

# THERMAL PROPERTIES OF THE HEAVY AXIAL VECTOR QUARKONIA

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Using the additional operators coming up at finite temperature, we calculate the masses and decay constants of the  $P$  wave heavy axial-vector  $\chi_{b1}$  and  $\chi_{c1}$  quarkonia in the framework of thermal QCD sum rules. In the calculations, we take into account the perturbative two loop order  $\alpha_s$  corrections and nonperturbative effects up to the dimension four condensates. It is observed that the masses and decay constants almost remain unchanged with respect to the variation of the temperature up to  $T \simeq 100$  MeV, however after this point, the decay constants decrease sharply and approach approximately to zero at critical temperature. The decreasing in values of the masses is also considerable after  $T \simeq 100$  MeV.

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## 1. INTRODUCTION

Investigation of the in medium properties of heavy mesons such as bottomonium ( $\bar{b}b$ ) and charmonium ( $\bar{c}c$ ) are of considerable interest for hadron physics to date. These quarkonia play an important role in obtaining information on the restoration of the spontaneously broken chiral symmetry in a nuclear medium and understanding quark gluon plasma (QGP) as a new phase of hadronic matter. The investigations of hadrons can also provide us with substantial knowledge on the nonperturbative QCD and interaction of quarks and gluons with QCD vacuum [1]. A plenty of theoretical works have also been dedicated to study the thermal behavior of hadronic parameters as well as QCD degrees of freedom (for some of them and discussion on the QGP phase see for instance [2–29]).

Hadrons are formed in a region of energy very far from the perturbative region, hence to calculate their parameters we need to have some nonperturbative approaches. The QCD sum rules as one of the most attractive, applicable and powerful techniques has been in the focus of much attention during last 32 years. This approach at zero temperature proposed in [1] and have applied to many decay channels in this period giving results in a good consistency with the existing experimental

data as well as lattice QCD calculations. This method then was extended to finite temperature QCD in [2]. There are two new aspects in this extension compared to the case at zero temperature [3–5], namely interaction of the particles in the medium with the currents requiring modification of the hadronic spectral density as well as the breakdown of the Lorentz invariance *via* the choice of reference frames. Because of residual  $O(3)$  symmetry at finite temperature, more operators with the same dimensions come out in the operator product expansion (OPE) compared to that of vacuum.

The purpose of this paper is to calculate the masses and decay constants of the  $P$  wave heavy axial vector  $\chi_{b1}$  and  $\chi_{c1}$  mesons in the framework of the thermal QCD sum rules. In our calculations, we use thermal propagator containing new non-perturbative contributions appearing at finite temperature, and take into account the perturbative two-loop order corrections to the correlation function [1, 30]. We use the expressions of the temperature-dependent energy-momentum tensor obtained *via* Chiral perturbation theory [31] and lattice QCD [9, 10] as well as temperature-dependent gluon condensates and continuum threshold to obtain the behavior of the masses and decay constants of these mesons in terms of temperature.

## 2. THERMAL QCD SUM RULE FOR $P$ WAVE HEAVY AXIAL VECTOR QUARKONIA

In order to extract the sum rules for the masses and decay constants of the heavy axial vector  $\chi_{b1}$  and  $\chi_{c1}$  mesons at finite temperature, we start considering the following two-point thermal correlation function:

$$\Pi_{\mu\nu}(q, T) = i \int d^4x e^{iq \cdot x} \langle \mathcal{T}(J_\mu(x) J_\nu^\dagger(0)) \rangle, \quad (1)$$

where,  $J_\mu(x) =: \bar{Q}(x) \gamma_\mu \gamma_5 Q(x) :$  with  $Q = b$  or  $c$  is the interpolating current of heavy axial vector meson,  $T$  is temperature and  $\mathcal{T}$  indicates the time ordering product. The thermal average of any operator  $O$  is defined as

$$\langle O \rangle = \frac{\text{Tr}(e^{-\beta H} O)}{\text{Tr}(e^{-\beta H})}, \quad (2)$$

where  $H$  is the QCD Hamiltonian and  $\beta = 1/T$ .

According to the general philosophy of the QCD sum rules formalism, the above correlation function can be calculated in two different ways. Once, in terms of QCD degrees of freedom by the help of OPE called the theoretical or QCD side. The OPE incorporates the effects of the QCD vacuum through an infinite series of condensates of increasing mass dimensions. The second, in terms of hadronic parameters called the physical or phenomenological side. Matching then these two representations, we find sum rules for the physical observables under consideration.

To suppress the contribution of the higher states and continuum, we apply Borel transformation as well as continuum subtractions. In the following, we calculate the correlation function in two aforesaid windows.

### 2.1. THE PHENOMENOLOGICAL SIDE

Technically, to obtain the physical or phenomenological side of the correlation function, we insert a complete set of intermediate hadronic states with the same quantum numbers as the interpolating current  $J_\mu(x)$  into the correlation function. After performing the four-integral over  $x$  and isolating the ground state contribution, we get

$$\Pi_{\mu\nu}(q) = \frac{f_A^2 m_A^2}{m_A^2 - q^2} \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) + \dots, \quad (3)$$

where the  $f_A$  and  $m_A$  are decay constant and mass of the heavy axial vector meson, respectively. The dots in the above equation stand for the contribution of the excited heavy axial vector states and continuum. In deriving the Eq. (3), we have defined the decay constant  $f_A$  by the matrix element of the current  $J_\mu$  between the vacuum and the mesonic state in the following manner:

$$\langle 0 | J_\mu | A(q, \lambda) \rangle = f_A m_A \varepsilon_\mu^{(\lambda)}, \quad (4)$$

where  $\varepsilon_\mu$  is the four-polarization vector. We have also used the summation over polarization vectors as

$$\sum_\lambda \varepsilon_\mu^{(\lambda)*} \varepsilon_\nu^{(\lambda)} = -(g^{\mu\nu} - q_\mu q_\nu / m_A^2). \quad (5)$$

### 2.2. THE QCD SIDE

In QCD side, the correlation function is calculated in deep Euclidean region where  $q^2 \ll -\Lambda_{QCD}^2$  via OPE where the short or perturbative and long distance or non-perturbative effects are separated, *i.e.*,

$$\Pi_{\mu\nu}^{QCD}(q, T) = \Pi_{\mu\nu}^p(q, T) + \Pi_{\mu\nu}^{np}(q, T). \quad (6)$$

The short distance contributions are calculated using the perturbation theory, while the long distance contributions are expressed in terms of the thermal expectation values of the quark and gluon condensates as well as thermal average of the energy density coming up at finite temperature.

In the rest frame of the medium for axial vector meson at rest, the correlation function in QCD side can be written in terms of the transverse and longitudinal components as

$$\Pi_{\mu\nu}(q) = \left( \frac{q_\mu q_\nu}{q^2} - g_{\mu\nu} \right) \Pi_t(q) + \frac{q_\mu q_\nu}{q^2} \Pi_l(q), \quad (7)$$

where the functions,  $\Pi_t(q)$  and  $\Pi_l(q)$  are found in terms of the total correlation function as

$$\begin{aligned}\Pi_t(q) &= \frac{1}{3} \left( \frac{q^\mu q^\nu}{q^2} - g^{\mu\nu} \right) \Pi_{\mu\nu}(q), \\ \Pi_l(q) &= \frac{1}{q^2} q^\mu q^\nu \Pi_{\mu\nu}(q).\end{aligned}\quad (8)$$

Here, we would like to mention that the transverse and longitudinal components are related to each other in the limit  $|\mathbf{q}| \rightarrow 0$ , hence it is enough to use one of them to obtain the thermal sum rules for the physical quantities under consideration. Here, we use the function  $\Pi_t(q)$  for this aim. It can be shown that this function for the fixed values of the  $|\mathbf{q}|$ , can be written as [18]:

$$\Pi_t(q_0^2, T) = \int_0^\infty dq_0'^2 \frac{\rho_t(q_0'^2, T)}{q_0'^2 + Q_0^2}, \quad (9)$$

where  $Q_0^2 = -q_0^2$ , and

$$\rho_t(q_0^2, T) = \frac{1}{\pi} \text{Im} \Pi_t(q_0^2, T) \tanh \frac{\beta q_0}{2}, \quad (10)$$

is the spectral density. We also should stress that the function  $\Pi_t(q_0^2, T)$  receives contributions from both annihilation and scattering parts (for more information see [27]). However, as we deal with the mesons containing quark and antiquark with the same masses, the scattering part gives zero and here we focus our attention to calculate only the annihilation part.

The thermal correlation function in QCD side is obtained from Eq. (1) contracting out all quark fields *via* Wick's theorem. As a result, we obtain the following expression in terms of thermal heavy quarks propagators:

$$\Pi_{\mu\nu}(q, T) = i \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \left[ \gamma_\mu S(k) \gamma_\nu S(k-q) \right]. \quad (11)$$

In real time thermal field theory, the function  $\Pi_t(q_0^2, T)$  can be expressed in  $2 \times 2$  matrix representation, the elements of which depend on only one analytic function. Hence calculation of the 11-component of such matrix is enough to get information on the dynamics of the corresponding two-point correlation function. The 11-component of the thermal quark propagator  $S(k)$  which is given as a sum of its vacuum expression and a term depending on the temperature is given as [32]:

$$S(k) = (\gamma^\mu k_\mu + m) \left( \frac{1}{k^2 - m^2 + i\varepsilon} + 2\pi i n(|k_0|) \delta(k^2 - m^2) \right), \quad (12)$$

where  $n(x) = [\exp(\beta x) + 1]^{-1}$  is the Fermi distribution function and  $m$  is the quark mass. Performing the integral over  $k_0$  in the  $\mathbf{q} = 0$  limit, we get the imaginary part

of the  $\Pi_t(q_0^2, T)$  as:

$$Im\Pi_t(q_0^2, T) = -\int \frac{d\mathbf{k}}{8\pi^2} \frac{4m^2 - 3q_0\omega + 2\omega^2}{\omega^2} [1 - 2n(\omega) + 2n^2(\omega)] \delta(q_0 - 2\omega), \quad (13)$$

where  $\omega = \sqrt{\mathbf{k}^2 + m^2}$ . After standard calculations, we get the following expression for the annihilation part of the spectral density:

$$\rho_{t,a}(s) = \frac{s}{4\pi^2} [v(s)]^3 \left[ 1 - 2n\left(\frac{\sqrt{s}}{2T}\right) \right], \quad (14)$$

where  $v(s) = \sqrt{1 - 4m^2/s}$ . As we previously mentioned, we take into account also the perturbative two-loop  $\alpha_s$  order correction to the spectral density. At zero temperature, it is given as [1, 30]:

$$\rho_{\alpha_s}(s) = \alpha_s \frac{s}{3\pi^3} \left[ \pi v^3 \left( \frac{\pi}{2v} - \frac{1+v}{2} \left( \frac{\pi}{2} - \frac{3}{\pi} \right) \right) + \left( P^A(v) - P(v) \right) \ln \frac{1+v}{1-v} + Q^A(v) - Q(v) \right], \quad (15)$$

where we have set  $v = v(s)$  and the functions  $P(v)$ ,  $Q(v)$ ,  $P^A(v)$  and  $Q^A(v)$  are given as:

$$\begin{aligned} P(v) &= \frac{5}{4}(1+v^2)^2 - 2, \\ Q(v) &= \frac{3}{2}v(1+v^2), \\ P^A(v) &= \frac{21}{32} + \frac{59}{32}v^2 + \frac{19}{32}v^4 - \frac{3}{32}v^6, \\ Q^A(v) &= -\frac{21}{16}v + \frac{30}{16}v^3 + \frac{3}{16}v^5. \end{aligned} \quad (16)$$

To get the thermal version of the above two-loop  $\alpha_s$  order correction, we replace the strong coupling  $\alpha_s$  by its temperature dependent lattice improved version given in [27] (for more details see also [6, 7]).

Our final task in this section is to calculate the nonperturbative part of the thermal correlation function. The nonperturbative part in our case can be written in terms of operators up to dimension four as:

$$\Pi_t^{np}(q_0^2, T) = C_1 \langle \bar{\psi}\psi \rangle + C_2 \langle G_{\mu\nu}^a G^{a\mu\nu} \rangle + C_3 \langle u\Theta u \rangle. \quad (17)$$

where,  $C_n(q^2)$  are as Wilson coefficients. As we also previously mentioned, at finite temperature the Lorentz invariance is broken by the choice of reference frame and new operators appear in the Wilson expansion above. The new four-dimension operator here is  $\langle u\Theta u \rangle$ , where  $\Theta^{\mu\nu}$  is the energy momentum tensor and  $u^\mu$  is the four-velocity of the heat bath and it is introduced to restore Lorentz invariance formally in the thermal field theory. In the rest frame of the heat bath, we have  $u^\mu = (1, 0, 0, 0)$

which leads to  $u^2 = 1$ . Note that in our calculations, we ignore the heavy quark condensate  $\langle \bar{\psi}\psi \rangle$  since it suppress by inverse powers of the heavy quark mass.

To proceed in calculation of the nonperturbative part, we use the nonperturbative part of the quark propagator in an external gluon field,  $A_\mu^a(x)$  in the Fock-Schwinger gauge,  $x^\mu A_\mu^a(x) = 0$ . In this gauge, the vacuum gluon field  $A_\mu^a(k')$  is written in terms of gluon field strength tensor in momentum space as follows:

$$A_\mu^a(k') = -\frac{i}{2}(2\pi)^4 G_{\rho\mu}^a(0) \frac{\partial}{\partial k'_\rho} \delta^{(4)}(k'), \quad (18)$$

where  $k'$  is the gluon momentum. Taking into account one and two gluon lines

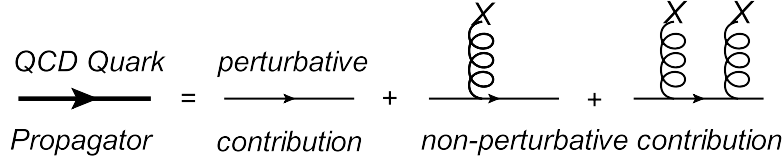


Fig. 1 – The quark propagator in the gluon background fields.

attached to the quark line as shown in Fig. 1, up to terms required for our calculations, the non-perturbative part of the temperature-dependent massive quark propagator is obtained as:

$$\begin{aligned} S^{aa'np}(k) = & -\frac{i}{4} g(t^c)^{aa'} G_{\kappa\lambda}^c \frac{1}{(k^2 - m^2)^2} \left[ \sigma_{\kappa\lambda} (\not{k} + m) + (\not{k} + m) \sigma_{\kappa\lambda} \right] \\ & + \frac{i g^2 \delta^{aa'}}{9 (k^2 - m^2)^4} \left\{ \frac{3m(k^2 + m \not{k})}{4} \langle G_{\alpha\beta}^c G^{c\alpha\beta} \rangle + \left[ m(k^2 - 4(k \cdot u)^2) \right. \right. \\ & \left. \left. + (m^2 - 4(k \cdot u)^2) \not{k} + 4(k \cdot u)(k^2 - m^2) \not{u} \right] \langle u^\alpha \Theta_{\alpha\beta}^g u^\beta \rangle \right\}, \end{aligned} \quad (19)$$

where  $\Theta_{\alpha\beta}^g$  is the traceless gluonic part of the energy-momentum tensor of the QCD.

Using the above expression and after straightforward but lengthy calculations, we get the following expression for the nonperturbative part:

$$\begin{aligned} \Pi_t^{np} = & \int_0^1 dx \left\{ \frac{g^2}{144q^2\pi^2(m^2 + q^2(-1+x)x)^4} \right. \\ & \times \left[ q^2(9A(-8q^6(-1+x)^4x^4 - m^2q^4(-1+x)^2x^2) \right. \\ & \times (-3 - 26x + 28x^2 - 4x^3 + 2x^4) + 3m^6(5 - 16x + 26x^2 - 20x^3 + 10x^4) \\ & \left. \left. + 2m^4q^2x(-6 + 3x + 16x^2 - 33x^3 + 30x^4 - 10x^5) \right) + B(-8m^2q^4(-1+x)^2 \right. \\ & \left. \times x^2(4 + 5x - 4x^2 - 2x^3 + x^4) - q^6(-1+x)^3x^3(6 + 19x - 17x^2 - 4x^3 + 2x^4) \right] \end{aligned}$$

$$\begin{aligned}
& -2m^6(29 - 92x + 150x^2 - 116x^3 + 58x^4) + 11m^4q^2x \\
& \times (6 - 11x + 8x^2 + x^3 - 6x^4 + 2x^5)) - 2B(-2q^6(-1 + x)^3x^3 \\
& \times (6 - 17x + 19x^2 - 4x^3 + 2x^4) - m^2q^4(-1 + x)^2x^2 \\
& \times (37 - 154x + 188x^2 - 68x^3 + 34x^4) + m^6(19 - 64x + 102x^2 - 76x^3 \\
& + 38x^4) - 4m^4q^2x(-6 + 47x - 116x^2 + 143x^3 - 102x^4 + 34x^5))(q \cdot u)^2 \Big\}, \tag{20}
\end{aligned}$$

where  $A = \frac{1}{24} \langle G_{\mu\nu}^a G^{a\mu\nu} \rangle + \frac{1}{6} \langle u^\alpha \Theta_{\alpha\beta}^g u^\beta \rangle$  and  $B = \frac{1}{3} \langle u^\alpha \Theta_{\alpha\beta}^g u^\beta \rangle$ .

### 2.3. THERMAL SUM RULES FOR PHYSICAL QUANTITIES

Now it is time to equate two different representations of the correlation function from physical and QCD sides and perform continuum subtraction to suppress the contribution of the higher states and continuum. As a result of this procedure we get the following sum rule including the temperature-dependent mass and decay constant:

$$\frac{f_A^2(T)Q_0^4}{[m_A^2(T) + Q_0^2] m_A^2(T)} = Q_0^4 \int_{4m^2}^{s_0(T)} \frac{[\rho_{t,a}(s) + \rho_{\alpha_s}(s)]}{s^2(s + Q_0^2)} ds + \Pi_t^{np}, \tag{21}$$

where  $s_0(T)$  is temperature-dependent continuum threshold and for simplicity, the temperature-dependent width of meson has been neglected. To further suppress the higher states and continuum contributions, we also apply the Borel transformation with respect to  $Q_0^2$  to both sides of the above sum rule. As a result we get,

$$f_A^2(T)m_A^2(T) \exp\left(-\frac{m_A^2(T)}{M^2}\right) = \int_{4m^2}^{s_0(T)} ds [\rho_{t,a}(s) + \rho_{\alpha_s}(s)] \exp\left(-\frac{s}{M^2}\right) + \hat{B}\Pi_t^{np}, \tag{22}$$

where the nonperturbative part in Borel scheme is obtained as:

$$\begin{aligned}
\hat{B}\Pi_t^{np} = & \int_0^1 dx \frac{1}{48M^6\pi^2(-1+x)^4x^4} \exp\left[\frac{m^2}{M^2(-1+x)x}\right] g^2 \\
& \times \left\{ 3A(8M^6(-1+x)^4x^4 + m^6(1-2x)^2(1-2x+2x^2)) \right. \\
& - m^2M^4(-1+x)^2x^2(-3-2x+4x^2-4x^3+2x^4) \\
& + m^4M^2x(3-14x+14x^2+13x^3-24x^4+8x^5)) \\
& - B(3m^6(1-2x)^2(1-2x+2x^2) + M^6(-1+x)^3x^3 \\
& \times (6-29x+31x^2-4x^3+2x^4) - m^2M^4(-1+x)^2 \\
& \times x^2(-4-29x+43x^2-28x^3+14x^4) \\
& \left. + m^4M^2x(8-35x+22x^2+69x^3-96x^4+32x^5)) \right\}. \tag{23}
\end{aligned}$$

Here  $M^2$  is the Borel mass parameter. Considering Eq. (22), the mass squared of the heavy axial vector meson alone can be obtained as:

$$m_A^2(T) = \frac{G(T)}{F(T)}, \quad (24)$$

where,

$$F(T) = \int_{4m^2}^{s_0(T)} ds \left[ \rho_{t,a}(s) + \rho_{\alpha_s}(s) \right] \exp\left(-\frac{s}{M^2}\right) + \widehat{B}\Pi_t^{np}, \quad (25)$$

and

$$G(T) = M^4 \frac{d}{dM^2} F(T). \quad (26)$$

#### 2.4. NUMERICAL RESULTS

To numerically analyse the sum rules for mass and decay constant, we use the following temperature-dependent continuum threshold [8]:

$$s_0(T) = s_0 \left[ 1 - \left( \frac{T}{T_c^*} \right)^8 \right] + 4 m^2 \left( \frac{T}{T_c^*} \right)^8, \quad (27)$$

where  $T_c^* = 1.1 T_c = 0.176 \text{ GeV}$  with  $T_c$  being critical temperature and  $s_0$  is the continuum threshold at zero temperature. For the temperature-dependent gluon condensate we also use [9, 10]

$$\langle G^2 \rangle = \frac{\langle 0|G^2|0 \rangle}{\exp \left[ 12 \left( \frac{T}{T_c} - 1.05 \right) \right] + 1}. \quad (28)$$

For the thermal average of total energy density  $\langle \Theta \rangle$  we use both results: i) obtained in lattice QCD [9, 10]:

$$\langle \Theta \rangle = 2 \langle \Theta^g \rangle = 6 \times 10^{-6} \exp[80(T - 0.1)] (\text{GeV}^4), \quad (29)$$

where this parametrization is valid only in the region  $0.1 \text{ GeV} \leq T \leq 0.17 \text{ GeV}$ . ii) obtained *via* chiral perturbation theory [31]:

$$\langle \Theta \rangle = \langle \Theta_\mu^\mu \rangle + 3 p, \quad (30)$$

where  $\langle \Theta_\mu^\mu \rangle$  is trace of the total energy momentum tensor and  $p$  is pressure. They are given as:

$$\begin{aligned} \langle \Theta_\mu^\mu \rangle &= \frac{\pi^2}{270} \frac{T^8}{F_\pi^4} \ln \left( \frac{\Lambda_p}{T} \right), \\ p &= 3T \left( \frac{m_\pi T}{2\pi} \right)^{\frac{3}{2}} \left( 1 + \frac{15 T}{8 m_\pi} + \frac{105 T^2}{128 m_\pi^2} \right) \exp\left(-\frac{m_\pi}{T}\right), \end{aligned} \quad (31)$$

where  $\Lambda_p = 0.275 \text{ GeV}$ ,  $F_\pi = 0.093 \text{ GeV}$  and  $m_\pi = 0.14 \text{ GeV}$ .



We also use the values  $m_c = (1.3 \pm 0.05)$  GeV,  $m_b = (4.7 \pm 0.1)$  GeV and  $\langle 0 | \frac{1}{\pi} \alpha_s G^2 | 0 \rangle = (0.012 \pm 0.004)$  GeV<sup>4</sup> for quarks masses and gluon condensate at zero temperature. Finally, we should find the working region for the continuum threshold at zero temperature ( $s_0$ ) and Borel mass parameter ( $M^2$ ) such that the physical observables are weakly depend on these parameters according to the standard criteria of the QCD sum rules. The continuum threshold,  $s_0$  is not totally arbitrary and it is correlated to the energy of the first excited state of the heavy axial vector meson. Our numerical calculations lead to the intervals  $s_0 = (106 - 110)$  GeV<sup>2</sup> and  $s_0 = (15 - 17)$  GeV<sup>2</sup> for the  $\chi_{b1}$  and  $\chi_{c1}$  heavy axial mesons, respectively. The working region for the Borel mass parameter is calculated requiring that not only the contributions of the higher states and continuum are efficiently suppressed but also the contributions of the operators with higher dimensions are ignorable. We get the working regions  $10 \text{ GeV}^2 \leq M^2 \leq 35 \text{ GeV}^2$  and  $5 \text{ GeV}^2 \leq M^2 \leq 25 \text{ GeV}^2$  respectively for the  $\chi_{b1}$  and  $\chi_{c1}$  channels.

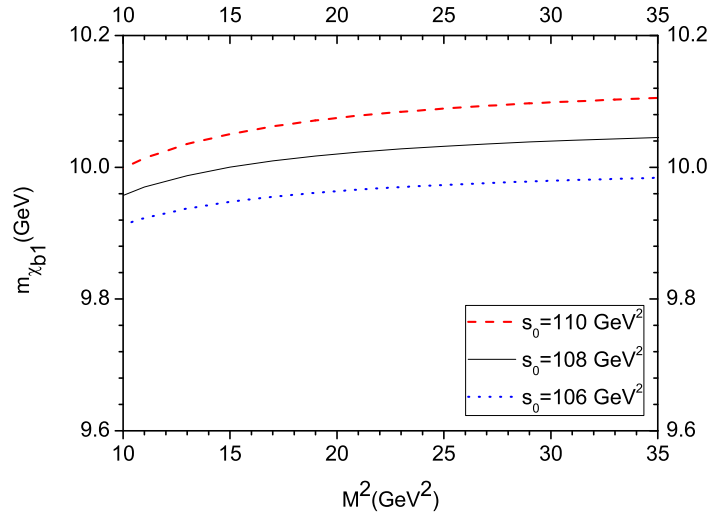


Fig. 2 – Dependence of the mass of the  $\chi_{b1}$  meson on the Borel parameter  $M^2$  at zero temperature.

Using the above obtained working regions for auxiliary parameters together with the other inputs, we plot the dependence on the Borel parameter  $M^2$  of the masses and decay constants of the heavy axial  $\chi_{b1}$  and  $\chi_{c1}$  quarkonia at zero temperature in Figs. (2-5). From these figures, we see that the results weakly depend on the auxiliary parameters in their working regions. The numerical results for the masses and decay constants of the heavy axial vector mesons under consideration are depicted in tables 1 and 2. We also compare the obtained results with the exper-

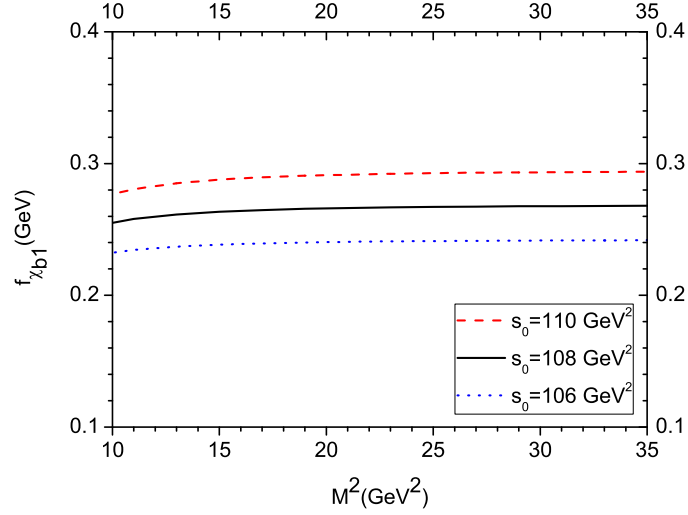


Fig. 3 – Dependence of the decay constant of the  $\chi_{b1}$  meson on the Borel parameter  $M^2$  at zero temperature.

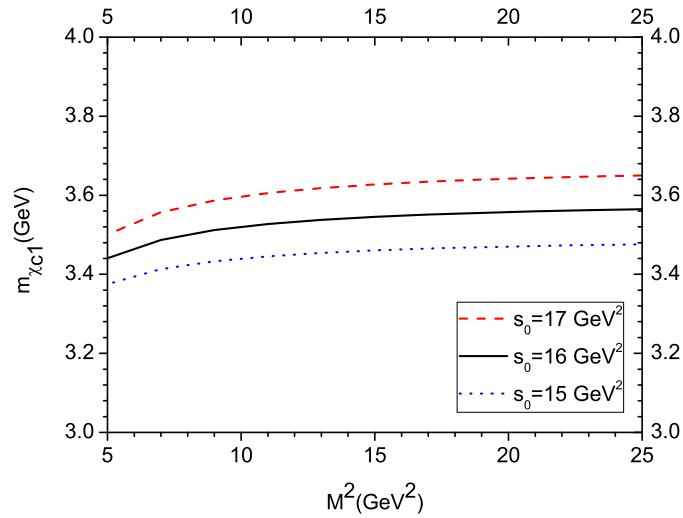


Fig. 4 – Dependence of the mass of the  $\chi_{c1}$  meson on the Borel parameter  $M^2$  at zero temperature.

imental values in the same tables. From table 1 we see a good consistency of our results with the experimental data. The errors in the results of our work belong to the uncertainties in calculation of the working regions for auxiliary parameters as well

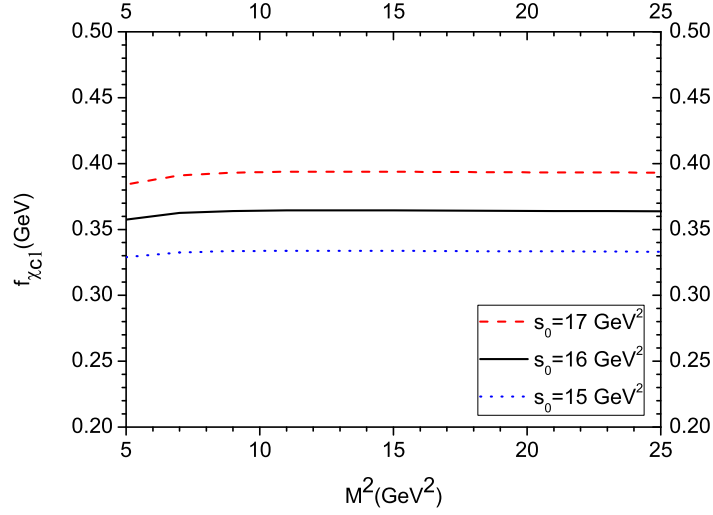


Fig. 5 – Dependence of the decay constant of the  $\chi_{c1}$  meson on the Borel parameter  $M^2$  at zero temperature.

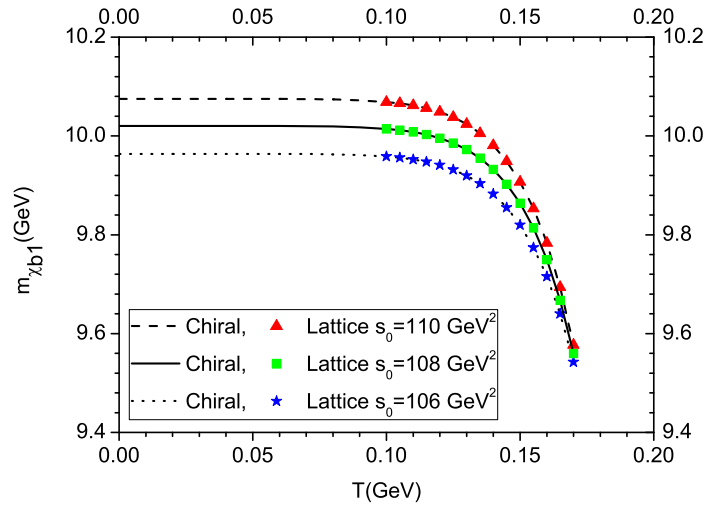


Fig. 6 – Dependence of the mass of the  $\chi_{b1}$  meson on temperature at  $M^2 = 20$  GeV<sup>2</sup>.

as those coming from other inputs.

At the end of this section we would like to discuss the behavior of the decay constants and masses of the heavy axial quarkonia under consideration in terms of

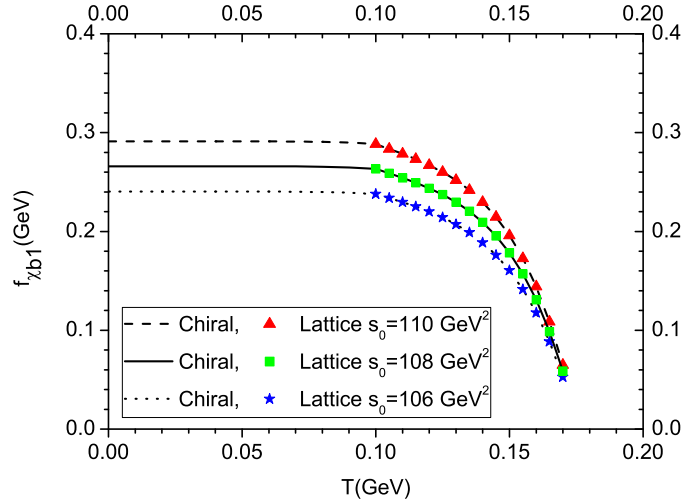


Fig. 7 – Dependence of the decay constant of the  $\chi_{b1}$  meson on temperature at  $M^2 = 20 \text{ GeV}^2$ .

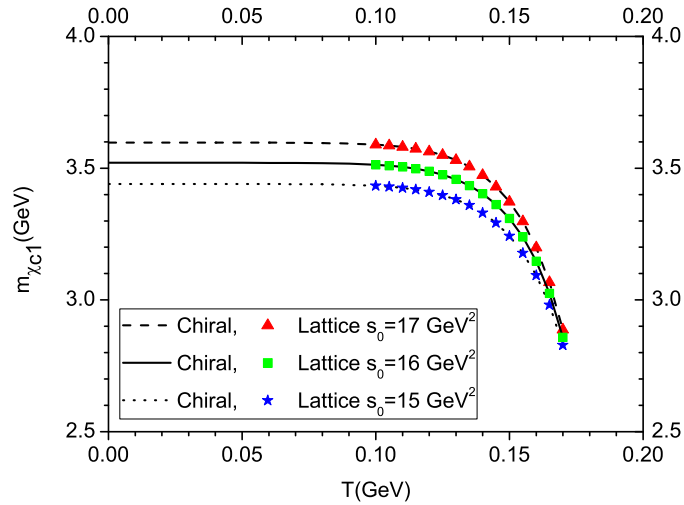


Fig. 8 – Dependence of the mass of the  $\chi_{c1}$  meson on temperature at  $M^2 = 10 \text{ GeV}^2$ .

temperature. We depict the variations of these quantities versus temperature in Figs. (6-9). From these figures, we see that the masses and decay constants remain unchanged with the variation of temperature up to  $T \cong 100 \text{ MeV}$ . After this point they start to decrease increasing the temperature. At deconfinement or critical tempera-

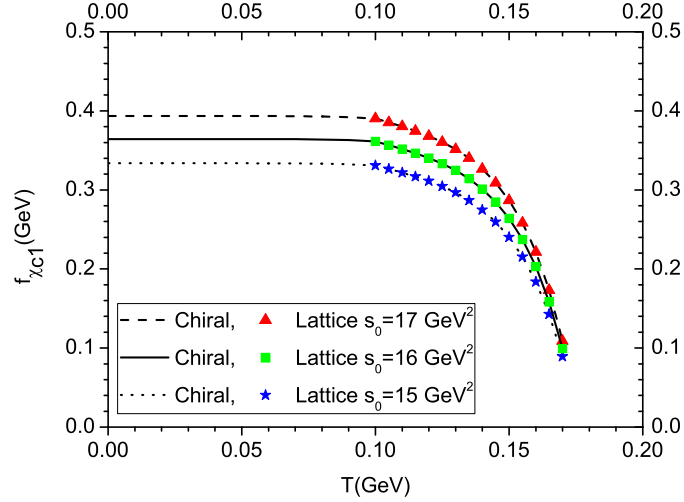


Fig. 9 – Dependence of the decay constant of the  $\chi_{c1}$  meson on temperature at  $M^2 = 10 \text{ GeV}^2$ .

Table 1

Values for the masses of the heavy axial  $\chi_{b1}$  and  $\chi_{c1}$  quarkonia at zero temperature.

	$m_{\chi_{c1}}$ (GeV)	$m_{\chi_{b1}}$ (GeV)
Present Work	$3.52 \pm 0.11$	$9.96 \pm 0.26$
Experiment [33]	$3.51066 \pm 0.00007$	$9.89278 \pm 0.00026 \pm 0.00031$

Table 2

Values for the decay constants of the heavy axial  $\chi_{b1}$  and  $\chi_{c1}$  quarkonia at zero temperature.

	$f_{\chi_{c1}}$ (MeV)	$f_{\chi_{b1}}$ (MeV)
Present Work	$344 \pm 27$	$240 \pm 12$

ture, the decay constants decrease about (73-78)%, while the masses are decreased about 4%, and 19% for  $\chi_{b1}$  and  $\chi_{c1}$  states, respectively. The sharp decreasing in the values of the decay constants near the deconfinement temperature can be considered as a signal for existing the QGP as the new phase of hadronic matter.

## 2.5. CONCLUSION

The determination of the thermal properties of heavy axial vector mesons can play essential role in understanding the vacuum properties of the non-perturbative QCD. In this paper, we calculated the masses and decay constants of the heavy axial vector  $\chi_{b1}$  and  $\chi_{c1}$  quarkonia in the framework of the thermal QCD sum rules.

In particular, we used the quark propagator at finite temperature and calculated the annihilation and scattering parts of the spectral densities for axial vector currents. In our calculations we also used the results of the energy density for the interval  $T = (0 - 170) \text{ MeV}$  obtained *via* Chiral perturbation theory [31] as well as the values of the energy density and gluon condensates obtained in the region  $T = (100 - 170) \text{ MeV}$  *via* lattice QCD [9, 10]. We observed that the values of the decay constants decrease considerably near to the critical or deconfinement temperature comparing to their values in vacuum. Our analysis also shows that the orders of decreasing in the values of the decay constants and masses are comparable with those of the scalar quarkonia channels [25], but they are considerably higher than those of the pseudoscalar and vector quarkonia channels [26, 27]. Our calculations also show that the perturbative two-loop order corrections are significantly important in this channel compared to the other quarkonia channels.

Our results at zero temperature as well as the behavior of the masses and decay constants with respect to the temperature can be checked in future experiments. Also the temperature dependence of the considered quantities can be used in analysis of the heavy ion collision experiments.

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#### REFERENCES

1. M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. *B* **147**, 385 (1979); Nucl. Phys. *B* **147**, 448 (1979).
2. A.I. Bochkarev, M.E. Shaposhnikov, Nucl. Phys. *B* **268**, 220 (1986).
3. E.V. Shuryak, Rev. Mod. Phys. **65**, 1 (1993).
4. T. Hatsuda, Y. Koike, S.H. Lee, Nucl. Phys. *B* **394**, 221 (1993).
5. S. Mallik, Phys. Lett. *B* **416**, 373 (1998).
6. K. Morita, S.H. Lee, Phys. Rev. *C* **77**, 064904 (2008); Phys. Rev. *D* **82**, 054008 (2010).
7. O. Kaczmarek, F. Karsch, F. Zantow, P. Petreczky, Phys. Rev. *D* **70**, 074505 (2004).
8. C.A. Dominguez, M. Loewe, J.C. Rojas, Y. Zhang, Phys. Rev. *D* **81**, 014007 (2010).
9. M. Cheng *et al.*, Phys. Rev. *D* **77**, 014511 (2008).
10. D.E. Miller, Phys. Rept. **443**, 55 (2007).
11. N. Brambilla, J. Ghiglieri, A. Vairo, Phys. Rev. *D* **78**, 014017 (2008).
12. N. Brambilla, M.A. Escobedo, J. Ghiglieri, J. Soto, A. Vairo, JHEP **1009**, 038 (2010).
13. Y. Aoki *et al.*, Nature **443**, 675 (2006).
14. Y. Aoki *et al.*, JHEP **0601**, 089 (2006).
15. S. Borsanyi *et al.*, JHEP **1011**, 077 (2010).
16. S. Borsanyi *et al.*, JHEP **1009**, 073 (2010).

17. Y. Aoki *et al.*, Phys. Lett. B **643** 46 (2006);  
JHEP *0906* 088 (2009).
18. S. Mallik, K. Mukherjee, Phys. Rev. D **58**, 096011 (1998);  
Phys. Rev. D *61*, 116007 (2000).
19. S. Mallik, S. Sarkar, Phys. Rev. D **66**, 056008 (2002).
20. E.V. Veliev, J. Phys. G: Nucl. Part. Phys. **G 35**, 035004 (2008);  
E.V. Veliev, T.M. Aliev, J. Phys. G: Nucl. Part. Phys., **G 35**, 125002 (2008).
21. C.A. Dominguez, M. Loewe, J.C. Rojas, JHEP **08**, 040 (2007).
22. E.V. Veliev, G. Kaya, Eur. Phys. J. C *63*, 87 (2009);  
Acta Phys. Polon. B *41*, 1905 (2010).
23. F. Klingl, S. Kim, S.H. Lee, P. Morath, W. Weise, Phys. Rev. Lett. **82**, 3396 (1999).
24. G. Aarts and J.M. Martinez Resco, Nucl. Phys. **B 93**, 726 (2005);  
JHEP **0204**, 053 (2002).
25. E.V. Veliev, H. Sundu, K. Azizi, M. Bayar, Phys. Rev. D **82**, 056012 (2010).
26. E.V. Veliev, K. Azizi, H. Sundu, N. Akşit, J. Phys. G **39**, 015002 (2012).
27. E. Veli Veliev, K. Azizi, H. Sundu, G. Kaya, A. Turkan, Eur. Phys. J. A **47**, 110 (2011).
28. K. Azizi, N. Er, Phys. Rev. D **81**, 096001 (2010).
29. E. Veli Veliev, K. Azizi, H. Sundu, G. Kaya, PoS (Confinement X) **339** (2012).
30. L.J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rep. **127**, 1 (1985).
31. P. Gerber, H. Leutwyler, Nucl. Phys. **B321**, 387 (1989).
32. A. Das, *Finite Temperature Field Theory* (World Scientific, 1999).
33. K. Nakamura *et al.* (Particle Data Group), J. Phys. G **37**, 075021 (2010).