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Thermal properties of $D_2^*(2460)$ and $D_{s2}^*(2573)$ tensor mesons using QCD sum rules

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Abstract

We investigate the masses and decay constants of the heavy–light $D_2^*(2460)$ and $D_{s2}^*(2573)$ tensor mesons in the framework of thermal QCD sum rules. Taking into account the additional operators arising at finite temperature, we evaluate the Wilson expansion for the two-point correlation function associated with these mesons. We observe that the values of the masses and decay constants decrease considerably at near to the critical temperature. The decay constants attain roughly to 25% of their values in vacuum, while the masses decrease about 39% and 37% in D_2^* and D_{s2}^* channels, respectively.

Keywords: QCD sum rules, finite-temperature field theory, charmed mesons

(Some figures may appear in colour only in the online journal)

1. Introduction

During the last few decades, many tensor mesons have been observed by different experiments [1–7]. The investigation of these particles is one of the most interesting problems in hadron physics both theoretically and experimentally. In the literature, there are few theoretical works devoted to the analysis of the properties of the tensor mesons compared to the scalar, pseudoscalar, vector and axial-vector mesons. The study of parameters of tensor mesons and their comparison with the experimental results can give useful information on their nature and internal structure. Moreover, the investigation of these particles can be useful for understanding the non-perturbative dynamics as well as the vacuum structure of QCD.

The observation of charmed $D_2^*(2460)$ and $D_{s2}^*(2573)$ states both with quantum numbers $J^P = 2^+$, were reported twenty years ago [4–6] and confirmed by the LHCb collaboration in 2011 [8]. The properties of these mesons at zero temperature have been recently studied in

[9, 10]. In this paper, we investigate the thermal properties of these particles, which can be used in analysis of the results of the heavy ion collisions held at different experiments.

The study of parameters of mesons at finite temperature requires some thermal non-perturbative approaches. One of the most attractive and applicable tools in this respect is the thermal QCD sum rules firstly suggested for investigation of hadronic parameters in vacuum [11] and later was extended to finite temperature and density [12]. This extension was based on some basic assumptions so that the Wilson expansion and the quark-hadron duality approximation remain valid, but the vacuum condensates are replaced by their thermal expectation values. At finite temperature, the Lorentz invariance is broken by the choice of a preferred frame of reference and some new operators appear in the Wilson expansion [13–16]. To restore the Lorentz invariance in thermal field theory, the four-vector velocity of the medium is introduced. Making use of this velocity and the fermionic and gluonic parts of the energy-momentum tensor, a new set of four dimensional operators are constructed. The thermal QCD sum rule method has been widely used to investigate the medium properties of the light–light [17, 18], the heavy–light [19–21] and the heavy–heavy [22–27] systems mainly in recent years.

In the present work, in particular, we investigate the masses and decay constants of the D_2^* and D_{s2}^* tensor mesons in the framework of thermal QCD sum rules method. Taking into account the additional operators coming up at finite temperature, we calculate the thermal two-point correlation function and obtain the spectral densities in one loop approximation. In order to perform the numerical analysis, we use the fermionic part of the energy density obtained both from lattice QCD [28, 29] and Chiral perturbation theory [30]. We also use the temperature dependent continuum threshold [19] and investigate the sensitivity of the masses and decay constants to the temperature.

The paper is organized as follows. In section 2 we evaluate the Wilson expansion for the two-point correlation function and derive the thermal QCD sum rules for the masses and decay constants of the D_2^* and D_{s2}^* tensor states. In section 3 we present our numerical calculations and discuss the obtained results.

2. Theoretical framework

To calculate the masses and decay constants of the D_2^* and D_{s2}^* tensor mesons in the framework of the thermal QCD sum rules, we start with the following thermal correlation function:

$$\Pi_{\mu\nu,\alpha\beta}(q, T) = i \int d^4x e^{iq \cdot (x-y)} \langle \mathcal{T} [j_{\mu\nu}(x) \bar{j}_{\alpha\beta}(y)] \rangle_{|y=0}, \quad (1)$$

where $j_{\mu\nu}$ is the interpolating current of the tensor mesons, T is temperature and \mathcal{T} indicates the time ordering operator. As the interpolating current of the tensor mesons contains derivatives with respect to the space-time, after applying derivatives with respect to y we will set $y = 0$. The thermal average of any operator A in thermal equilibrium is defined as $\langle A \rangle = \frac{\text{Tr}(e^{-\beta H} A)}{\text{Tr}(e^{-\beta H})}$, where H is the QCD Hamiltonian and $\beta = 1/T$.

The interpolating current $j_{\mu\nu}$ for tensor mesons is written as

$$j_{\mu\nu}(x) = \frac{i}{2} [\bar{q}(x) \gamma_\mu \overleftrightarrow{\mathcal{D}}_\nu(x) c(x) + \bar{q}(x) \gamma_\nu \overleftrightarrow{\mathcal{D}}_\mu(x) c(x)], \quad (2)$$

where q is u (s) quark for D_2^* (D_{s2}^*) and $\overleftrightarrow{\mathcal{D}}_\mu(x)$ denotes the four-derivative with respect to x acting on the left and right, simultaneously. It is given as

$$\overleftrightarrow{\mathcal{D}}_\mu(x) = \frac{1}{2} [\overrightarrow{\mathcal{D}}_\mu(x) - \overleftarrow{\mathcal{D}}_\mu(x)], \quad (3)$$

where

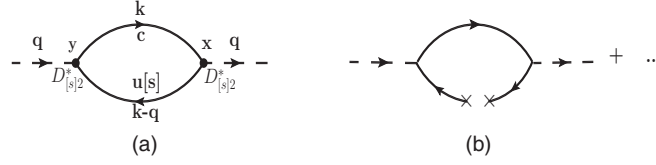


Figure 1. (a) Bare loop diagram (short-distance or perturbative contribution); (b) quark condensate diagram (the lowest dimension long-distance or non-perturbative contribution).

$$\begin{aligned}\vec{\mathcal{D}}_\mu(x) &= \vec{\partial}_\mu(x) - i\frac{g}{2}\lambda^a A_\mu^a(x), \\ \overleftarrow{\mathcal{D}}_\mu(x) &= \overleftarrow{\partial}_\mu(x) + i\frac{g}{2}\lambda^a A_\mu^a(x).\end{aligned}\quad (4)$$

Here, λ^a ($a = 1, 2, \dots, 8$) are the Gell–Mann matrices and $A_\mu^a(x)$ are the external gluon fields.

According to the basic idea in the QCD sum rule method, the aforementioned thermal correlation function can be calculated in two different ways: first, in terms of QCD degrees of freedom called theoretical or QCD side, and the second, in terms of hadronic parameters called the physical or phenomenological side. The correlation function in QCD side is calculated using the operator product expansion (OPE), where the short and long distance effects (see figure 1) are separated. The thermal QCD sum rules for the physical observables such as the masses and decay constants are obtained equating the coefficients of the same structure from both sides of the correlation function through a dispersion relation. Finally, the Borel transformation and continuum subtraction are performed in order to suppress the contributions of the higher states and continuum.

2.1. Correlation function in QCD representation

As previously mentioned, the correlation function in QCD side is evaluated via OPE in deep Euclidean region where the perturbative and non-perturbative contributions are separated. The perturbative part is calculated via perturbation theory using spectral representation, while the non-perturbative contributions are represented in terms of the thermal expectation values of the quark and gluon condensates as well as thermal average of the energy density. Putting the expression of the interpolating current and covariant four derivatives into correlation function in equation (1) and applying the Wick's theorem, we get

$$\begin{aligned}\Pi_{\mu\nu,\alpha\beta} &= -\frac{i}{16} \int d^4x e^{iq \cdot (x-y)} \{ \text{Tr} [\vec{\mathcal{D}}_\beta(y) S_q(y-x) \gamma_\mu \vec{\mathcal{D}}_\nu(x) S_c(x-y) \gamma_\alpha \\ &\quad - S_q(y-x) \gamma_\mu \vec{\mathcal{D}}_\nu(x) \vec{\mathcal{D}}_\beta(y) S_c(x-y) \gamma_\alpha \\ &\quad - \vec{\mathcal{D}}_\beta(y) \vec{\mathcal{D}}_\nu(x) S_q(y-x) \gamma_\mu S_c(x-y) \gamma_\alpha \\ &\quad + \vec{\mathcal{D}}_\nu(x) S_q(y-x) \gamma_\mu \vec{\mathcal{D}}_\beta(y) S_c(x-y) \gamma_\alpha] \\ &\quad + [\beta \leftrightarrow \alpha] + [v \leftrightarrow \mu] + [\beta \leftrightarrow \alpha, v \leftrightarrow \mu] \},\end{aligned}\quad (5)$$

where we kept only the full contracted terms. The normally ordered terms also give non-perturbative contributions which we take into account in the expressions of the propagators. The expressions for the heavy quark propagator $S_c(x-y)$ and the light quark propagator $S_q(x-y)$ in coordinate space, up to the terms considered in the present work, are given as

$$S_c^{ij}(x-y) = \frac{i}{(2\pi)^4} \int d^4k e^{-ik \cdot (x-y)} \left\{ \frac{\not{k} + m_c}{k^2 - m_c^2} \delta_{ij} + \dots \right\}, \quad (6)$$

and

$$S_q^{ij}(x-y) = i \frac{\not{x} - \not{y}}{2\pi^2(x-y)^4} \delta_{ij} - \frac{m_q}{4\pi^2(x-y)^2} \delta_{ij} - \frac{\langle \bar{q}q \rangle}{12} \delta_{ij} + \frac{i}{3} \left[(\not{x} - \not{y}) \left(\frac{m_q}{16} \langle \bar{q}q \rangle - \frac{1}{12} \langle u \Theta^f u \rangle \right) + \frac{1}{3} (u(x-y) \not{u} \langle u \Theta^f u \rangle) \right] \delta_{ij} + \dots, \quad (7)$$

where $\Theta_{\mu\nu}^f$ is the fermionic part of the energy momentum tensor and u_μ is the four-velocity of the heat bath. In the rest frame of the heat bath, $u_\mu = (1, 0, 0, 0)$ and $u^2 = 1$. Note that in our calculations we ignore the two-gluon condensate terms because of their small contributions (see also [31–33]).

The next step is to use the expressions of the propagators in equation (5) and apply the derivatives with respect to x and y . After setting $y = 0$, we get

$$\Pi_{\mu\nu,\alpha\beta} = \frac{1}{16} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_c^2} \int d^4x e^{i(q-k)\cdot x} \{ [\text{Tr} \Gamma_{\mu\nu,\alpha\beta}] + [\beta \leftrightarrow \alpha] + [v \leftrightarrow \mu] + [\beta \leftrightarrow \alpha, v \leftrightarrow \mu] \}, \quad (8)$$

where $\Gamma_{\mu\nu,\alpha\beta}$ is given by

$$\begin{aligned} \Gamma_{\mu\nu,\alpha\beta} = & k_\nu k_\beta \left[\frac{i \not{x}}{2\pi^2 x^4} + \frac{m_q}{4\pi^2 x^2} + \left(\frac{1}{12} + \frac{i m_q \not{x}}{48} \right) \langle \bar{q}q \rangle \right. \\ & - \left. \left(\frac{i \not{x}}{36} - \frac{i}{9} u \cdot x \not{u} \right) \langle u \Theta^f u \rangle \right] \gamma_\mu (\not{k} + m_c) \gamma_\alpha \\ & - i k_\nu \left[\frac{i}{2\pi^2} \left(\frac{\gamma_\beta}{x^4} - \frac{4x_\beta \not{x}}{x^6} \right) - \frac{m_q x_\beta}{2\pi^2 x^4} + \frac{i \gamma_\beta m_q}{48} \langle \bar{q}q \rangle \right. \\ & - \left. \left(\frac{i \gamma_\beta}{36} - \frac{i}{9} u_\beta \not{u} \right) \langle u \Theta^f u \rangle \right] \gamma_\mu (\not{k} + m_c) \gamma_\alpha \\ & + i k_\beta \left[\frac{i}{2\pi^2} \left(\frac{4x_\nu \not{x}}{x^6} - \frac{\gamma_\nu}{x^4} \right) + \frac{m_q x_\nu}{2\pi^2 x^4} - \frac{i m_q \gamma_\nu}{48} \langle \bar{q}q \rangle + \left(\frac{i \gamma_\nu}{36} - \frac{i}{9} u_\nu \not{u} \right) \langle u \Theta^f u \rangle \right] \\ & \times \gamma_\mu (\not{k} + m_c) \gamma_\alpha + \left[\frac{8i}{2\pi^2} \left(\frac{x_\beta x_\nu \not{x}}{x^8} + \frac{1}{2x^6} \left(\delta_\beta^\nu \not{x} - \gamma_\nu x_\beta + \gamma_\beta x_\nu \right) \right. \right. \\ & \left. \left. + \frac{\gamma_\nu x_\beta}{x^6} - \frac{4x_\beta x_\nu \not{x}}{x^8} \right) + \frac{\delta_\beta^\nu m_q}{2\pi^2 x^4} + \frac{2m_q x_\nu x_\beta}{\pi^2 x^6} \right] \\ & \times \gamma_\mu (\not{k} + m_c) \gamma_\alpha + [\beta \leftrightarrow \alpha] + [v \leftrightarrow \mu] + [\beta \leftrightarrow \alpha, v \leftrightarrow \mu]. \quad (9) \end{aligned}$$

2.2. Correlation function in phenomenological representation

To calculate the phenomenological side of the correlation function, a complete set of physical intermediate states having the same quantum numbers as the interpolating current is inserted into equation (1). After performing integral over x and putting $y = 0$, we obtain

$$\Pi_{\mu\nu,\alpha\beta}^{D_2^*(D_{s_2}^*)} = \frac{\langle 0 | j_{\mu\nu}(0) | D_2^*(D_{s_2}^*) \rangle \langle D_2^*(D_{s_2}^*) | \bar{j}_{\alpha\beta}(0) | 0 \rangle}{m_{D_2^*(D_{s_2}^*)}^2 - q^2} + \dots, \quad (10)$$

where dots indicate the contributions of the higher states and continuum. The matrix element $\langle 0 | j_{\mu\nu}(0) | D_2^*(D_{s_2}^*) \rangle$ can be written in terms of the decay constant $f_{D_2^*(D_{s_2}^*)}$ as

$$\langle 0 | j_{\mu\nu}(0) | D_2^*(D_{s_2}^*) \rangle = f_{D_2^*(D_{s_2}^*)} m_{D_2^*(D_{s_2}^*)}^3 \varepsilon_{\mu\nu}^{(\lambda)}, \quad (11)$$

where $\varepsilon_{\mu\nu}^{(\lambda)}$ is the polarization tensor. We use the summation over polarization tensors as

$$\sum_{\lambda} \varepsilon_{\mu\nu}^{(\lambda)} \varepsilon_{\alpha\beta}^{*(\lambda)} = \frac{1}{2} \eta_{\mu\alpha} \eta_{\nu\beta} + \frac{1}{2} \eta_{\mu\beta} \eta_{\nu\alpha} - \frac{1}{3} \eta_{\mu\nu} \eta_{\alpha\beta}, \quad (12)$$

where

$$\eta_{\mu\nu} = -g_{\mu\nu} + \frac{q_{\mu} q_{\nu}}{m_{D_2^*(D_{s_2}^*)}^2}. \quad (13)$$

Using the above expressions in equation (10), the final representation of the physical side is obtained as

$$\Pi_{\mu\nu,\alpha\beta}^{D_2^*(D_{s_2}^*)} = \frac{f_{D_2^*(D_{s_2}^*)}^2 m_{D_2^*(D_{s_2}^*)}^4}{m_{D_2^*(D_{s_2}^*)}^2 - q^2} \left\{ -\frac{1}{2} q_{\mu} q_{\alpha} g_{\nu\beta} \right\} + \text{other structures} + \dots, \quad (14)$$

where the explicitly written structure is used to extract the QCD sum rules for the physical quantities under consideration.

2.3. Thermal QCD sum rules for physical observables

To obtain the QCD sum rules for the masses and decay constants we need to calculate the perturbative and non-perturbative parts of the correlation function in momentum space then match the coefficients of the selected structure from both phenomenological and QCD sides. For this aim we write the perturbative part of the correlation function in QCD side in terms of a dispersion integral as

$$\Pi^{\text{pert}}(q, T) = \int \frac{ds \rho(s)}{s - q^2}, \quad (15)$$

where $\rho(s)$ is the spectral density and it is obtained via the imaginary part of the perturbative part of the thermal correlator

$$\rho(s) = \frac{1}{\pi} \text{Im}[\Pi^{\text{pert}}(s)]. \quad (16)$$

Following the procedures represented in [9, 10], and after lengthy calculations, we obtain the spectral densities corresponding to the tensor D_2^* and $D_{s_2}^*$ states as

$$\rho_{D_2^*}(s) = -\frac{N_c}{640\pi^2 s^4} (m_c^2 - s)^2 (8m_c^6 - 4m_c^4 s - m_c^2 s^2 + 2s^3), \quad (17)$$

and

$$\rho_{D_{s_2}^*}(s) = -\frac{N_c}{1920\pi^2 s^4} (m_c^2 - s) (24m_c^8 - 36m_c^6 s + 40m_c^5 m_s s + 9m_c^4 s^2 - 20m_c^3 m_s s^2 + 9m_c^2 s^3 + 10m_c m_s s^3 - 6s^4), \quad (18)$$

where $N_c = 3$ is the number of colors. From a similar way we calculate the non-perturbative contributions (see also [9, 10]).

The final task is to match the phenomenological and QCD sides of the correlation function in momentum space and apply Borel transformation with respect to $Q^2 = -q^2$. After continuum subtraction we get

$$f_{D_2^*(D_{s_2}^*)}^2(T) m_{D_2^*(D_{s_2}^*)}^4(T) e^{-m_{D_2^*(D_{s_2}^*)}^2(T)/M^2} = \int_{(m_q+m_c)^2}^{s_0(T)} ds \rho_{D_2^*(D_{s_2}^*)}(s) e^{-s/M^2} + \hat{B} \Pi_{D_2^*(D_{s_2}^*)}^{\text{non-pert}}, \quad (19)$$

where $s_0(T)$ is the temperature-dependent continuum threshold and M^2 is the Borel mass parameter. The function $\hat{B} \Pi_{D_2^*(D_{s_2}^*)}^{\text{non-pert}}$ shows the non-perturbative part of the QCD side in the Borel transformed scheme. It is given in D_2^* and $D_{s_2}^*$ channels as

$$\hat{B} \Pi_{D_2^*}^{\text{non-pert}} = -\frac{m_c \langle \bar{u}u \rangle}{48} e^{-m_c^2/M^2} + \frac{\langle u \Theta^f u \rangle}{72} e^{-m_c^2/M^2} - \frac{(-m_c^2 + M^2) \langle u \Theta^f u \rangle}{24M^2} e^{-m_c^2/M^2}, \quad (20)$$

and

$$\hat{B}\Pi_{D_2^*(D_{s_2}^*)}^{\text{non-pert}} = -\frac{m_c \langle \bar{s}s \rangle}{48} e^{-m_c^2/M^2} + \frac{m_s \langle \bar{s}s \rangle}{96} e^{-m_c^2/M^2} - \frac{m_s (-m_c^2 - M^2) \langle \bar{s}s \rangle}{96M^2} e^{-m_c^2/M^2} + \frac{\langle u\Theta^f u \rangle}{72} e^{-m_c^2/M^2} - \frac{(-m_c^2 + M^2) \langle u\Theta^f u \rangle}{24M^2} e^{-m_c^2/M^2}. \quad (21)$$

The temperature-dependent masses of the states under consideration are found as

$$m_{D_2^*(D_{s_2}^*)}^2(T) = \frac{\int_{(m_q+m_c)^2}^{s_0(T)} ds \rho_{D_2^*(D_{s_2}^*)}(s) s e^{-s/M^2} + \psi_{D_2^*(D_{s_2}^*)}^{\text{non-pert}}(M^2, T)}{\int_{(m_q+m_c)^2}^{s_0(T)} ds \rho_{D_2^*(D_{s_2}^*)}(s) e^{-s/M^2} + \hat{B}\Pi_{D_2^*(D_{s_2}^*)}^{\text{non-pert}}}, \quad (22)$$

where $\psi_{D_2^*(D_{s_2}^*)}^{\text{non-pert}}(M^2, T)$ is given by

$$\psi_{D_2^*(D_{s_2}^*)}^{\text{non-pert}}(M^2, T) = M^4 \frac{d}{dM^2} \hat{B}\Pi_{D_2^*(D_{s_2}^*)}^{\text{non-pert}}. \quad (23)$$

3. Numerical results and discussion

In this section we present our numerical results on the physical quantities under consideration and discuss their sensitivity to the temperature. We also compare the obtained numerical values at $T = 0$ with the existing experimental data [34] and those obtained from vacuum sum rules [9, 10]. For this aim, we use some input parameters as $m_s = 0.12$ GeV, $m_c = (1.27^{+0.07}_{-0.09})$ GeV [34], $\langle 0|\bar{u}u|0\rangle = -(0.24 \pm 0.01)^3$ GeV³ [35] and $\langle 0|\bar{s}s|0\rangle = 0.8\langle 0|\bar{u}u|0\rangle$ [36].

To proceed further, we use the fermionic part of the energy density obtained from both lattice QCD [28, 29] and Chiral perturbation theory [30]. The thermal average of the energy density obtained using the lattice QCD is expressed as

$$\langle \Theta \rangle = 2\langle \Theta^f \rangle = 6 \times 10^{-6} \exp[80(T - 0.1)], \quad (24)$$

where T is in the units of GeV and this parameterization is valid only in the region $0.1 \text{ GeV} \leq T \leq 0.175 \text{ GeV}$. Here we should mention that the total energy density has been calculated for $T \geq 0$ in Chiral perturbation theory, while it is available only for $T \geq 100$ MeV in lattice QCD [28, 29]. In the limit of low temperature Chiral perturbation, the thermal average of the energy density is written as [30]

$$\langle \Theta \rangle = \langle \Theta_\mu^\mu \rangle + 3p, \quad (25)$$

where $\langle \Theta_\mu^\mu \rangle$ is trace of the total energy momentum tensor and p is pressure. These quantities are given by

$$\langle \Theta_\mu^\mu \rangle = \frac{\pi^2 T^8}{270 F_\pi^4} \ln \left(\frac{\Lambda p}{T} \right), \quad (26)$$

and

$$p = 3T \left(\frac{m_\pi T}{2\pi} \right)^{\frac{3}{2}} \left(1 + \frac{15T}{8m_\pi} + \frac{105T^2}{128m_\pi^2} \right) \exp \left(-\frac{m_\pi}{T} \right). \quad (27)$$

In further analysis, we also use the light quark condensate at finite temperature. The temperature-dependent quark condensate obtained in Chiral perturbation theory [30, 37] can be written in a good approximation as

$$\langle \bar{q}q \rangle = \langle 0|\bar{q}q|0\rangle \left[1 - 0.4 \left(\frac{T}{T_c} \right)^4 - 0.6 \left(\frac{T}{T_c} \right)^8 \right]. \quad (28)$$

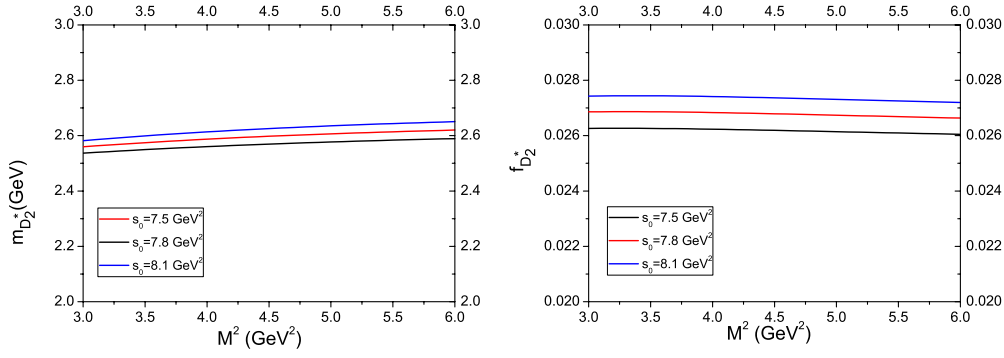


Figure 2. Variations of the mass and decay constant of the $D_2^*(2460)$ meson with respect to M^2 at fixed values of the continuum threshold and at $T = 0$.

where $T_c = 0.175$ GeV [38] is the critical temperature. The continuum threshold also depends on the temperature and it is given in terms of the quark condensate by [19]

$$s_0(T) = s_0 \frac{\langle \bar{q}q \rangle}{\langle 0 | \bar{q}q | 0 \rangle} \left(1 - \frac{(m_c + m_q)^2}{s_0} \right) + (m_c + m_q)^2, \quad (29)$$

where s_0 in the right hand side is the hadronic threshold at zero temperature, i.e., $s_0 = s(T = 0)$. The continuum threshold is not totally arbitrary but it depends on the energy of the first excited state with the same quantum numbers as the chosen interpolating current. According to the standard procedure in QCD sum rule approach the working region for this parameter is chosen such that the variations of the results with respect to this parameter in the chosen Borel window are weak. We choose the intervals $s_0 = (7.8 \pm 0.3)$ GeV² and $s_0 = (9.1 \pm 0.3)$ GeV² for the continuum threshold in the D_2^* and D_{s2}^* channels, respectively. Our analysis show that the dependences of the results on this parameter are very weak in these intervals.

From the sum rules for the physical quantities in the previous section it is clear that they also include an auxiliary Borel parameter M^2 which we shall also find its working region. The working region for the Borel parameter is found such that not only the contributions of the higher states and continuum are suppressed but also the perturbative part exceeds the non-perturbative contributions and the contributions of the higher dimensional operators are small, i.e., the OPE converges. As a result we obtain the interval $3 \text{ GeV}^2 \leq M^2 \leq 6 \text{ GeV}^2$ for the working region of Borel mass. Our numerical results show that the contribution of the higher states and continuum are approximately 10% of the total dispersion integral in the selected regions for the auxiliary parameters. To see how the results depend on the Borel mass parameter, we plot the dependences of the masses and decay constants of the mesons under consideration versus M^2 for different values of the continuum threshold at $T = 0$ in figures 2 and 3. From these figures we see that the results are practically independent from the Borel mass parameter for the aforesaid Borel working region and the selected structure. Our numerical analysis show also that the perturbative and non-perturbative parts overall constitute roughly 68% and 32% of the total ground state contribution, respectively. Moreover, the energy density constitutes about 10% of the total non-perturbative contribution in the working region of the Borel mass parameter. The rest contribution in the non-perturbative part comes from the quark condensate which also depend on the temperature according to equation (28).

Making use of all inputs we depict the variations of the masses and decay constants of the tensor mesons under consideration with respect to temperature in figures 4 and 5. From these figures we read that the results remain approximately unchanged up to 0.1 GeV, however, after this point, they start to diminish and fall considerably near to the critical temperature. These

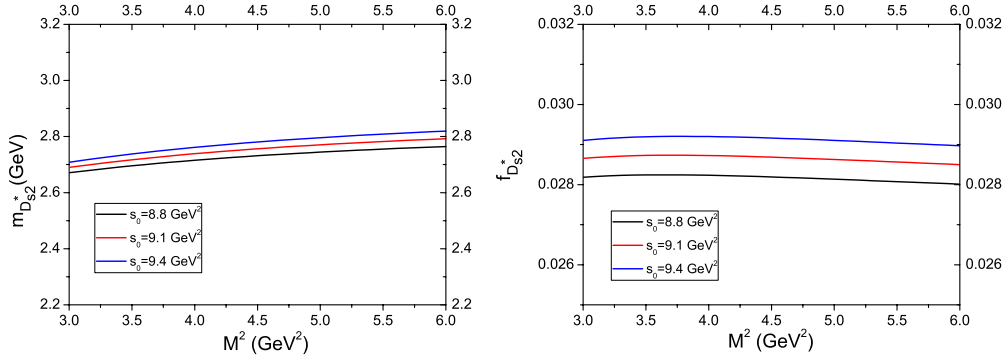


Figure 3. Variations of the mass and decay constant of the $D_{s_2}^*$ (2573) meson with respect to temperature at M^2 at fixed values of the continuum threshold and at $T = 0$.

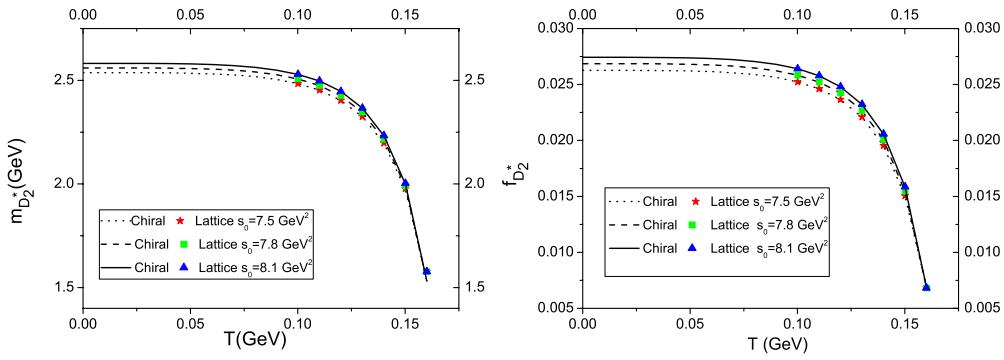


Figure 4. Variations of the mass and decay constant of the D_2^* (2460) meson with respect to temperature at $M^2 = 3 \text{ GeV}^2$.

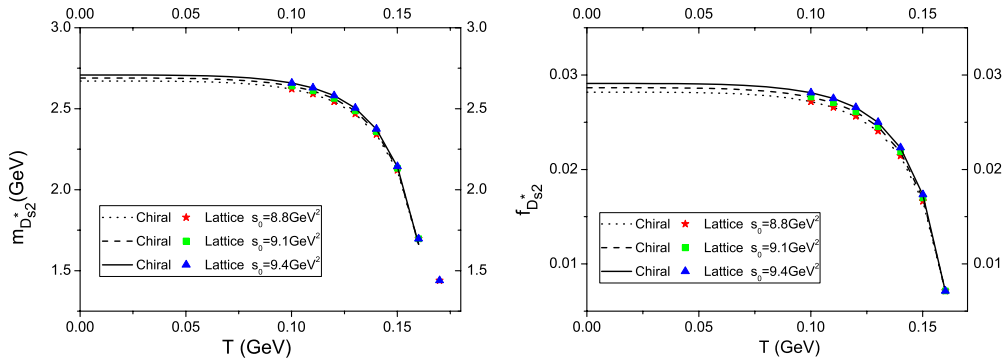


Figure 5. Variations of the mass and decay constant of the $D_{s_2}^*$ (2573) meson with respect to temperature at $M^2 = 3 \text{ GeV}^2$.

figures depict that the decay constants in both the Chiral and lattice parameterizations of the thermal average of the energy density reach roughly to 25% of their values in vacuum, while the masses decrease about 39% and 37% for D_2^* and $D_{s_2}^*$ states, respectively. Our results at finite temperature indicate also that the values of the physical quantities depend very weakly on the continuum threshold (s_0) such that near to the critical temperature the results became

Table 1. Values of the masses and decay constants of the tensor D_2^* and D_{s2}^* mesons at $T = 0$.

	Present work	Experiment [34]	Vacuum sum rules
$m_{D_2^*}^{(2460)}$ (GeV)	2.55 ± 0.46	2.4626 ± 0.0007	2.53 ± 0.45 [9]
$f_{D_2^*}^{(2460)}$	0.027 ± 0.013	–	0.0228 ± 0.0068 [9]
$m_{D_{s2}^*}^{(2573)}$ (GeV)	2.69 ± 0.48	2.5719 ± 0.0008	2.55 ± 0.44 [10]
$f_{D_{s2}^*}^{(2573)}$	0.029 ± 0.014	–	0.023 ± 0.011 [10]

practically independent of the continuum threshold at fixed value of the Borel mass parameter. It is also seen from these figures that the two Chiral and lattice parameterizations of the thermal average of the energy density lead to exactly the same results after $T = 0.1$ GeV.

Our final task is to compare our results in the limit $T \rightarrow 0$ with those previously obtained using vacuum sum rules as well as existing experimental data. This comparison is made in table 1. The errors quoted in this table for our results are due to the uncertainties in determinations of the working regions for the continuum threshold and Borel mass parameter as well as those coming from the errors of other input parameters. From this table we see that, within the uncertainties, our predictions on the masses of the tensor mesons are consistent with the experimental data as well as the vacuum sum rules predictions [9, 10] with a good approximation. Our results on the decay constants are also roughly consistent with those of [9, 10] within the errors. Our results on the leptonic decay constants can be checked in future experiments.

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