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More about the B and D mesons in nuclear matter

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Abstract We calculate the shifts in decay constants of the pseudoscalar B and D mesons in nuclear medium in the frame work of QCD sum rules. We write those shifts in terms of the B–N and D–N scattering lengths and an extra phenomenological parameter entered to calculations. Computing an appreciate forward scattering correlation function, we derive the QCD sum rules for the B–N and D–N scattering lengths and the extra phenomenological parameter in terms of various operators in nuclear medium. We numerically find the values of the shifts in the decay constants compared to their vacuum values. Using the sum rules obtained, we also determine the shifts in the masses of these particles due to nuclear matter and compare the results obtained with the previous predictions in the literature.

1 Introduction

Study the in-medium properties of hadrons can help us not only better understand the perturbative and non-perturbative natures of QCD, but also can play crucial role in analyzing the results of heavy ion collision experiments as well as understanding the internal structures of the dense astrophysical objects like neutron stars. From the experimental side, there have been a lot of experiments such as CEBAF and RHIC etc. focused on the study of the properties of hadrons in nuclear medium. The FAIR and CBM Collaborations intend to study the in-medium properties of different hadrons including the charmed mesons. The PANDA Collaboration also aims to focus on the study of the properties of hadrons in charm sector [1–4].

Along with the experimental progress, there were many theoretical works devoted to the study of the in-medium properties of hadrons. The basic properties of the nuclear matter are determined in [5]. Some finite-density problems and the saturation properties of nuclear matter are studied in [6–9]. In series of papers [10–12], the authors have studied the effects of nuclear matter on the masses of the nucleons. In [13], the ρ, ω and ϕ mesons–nucleon scattering lengths and their mass shifts in nuclear medium are investigated via QCD sum rules. [14] applies the same method to investigate the mass modification of D-meson at finite density. In [15], the authors expand the work of [14] to study the mass shift of also B meson in nuclear matter. The in-medium mass modification of the scalar charm meson is investigated in [16], which is then extended to include also the mass modification of the scalar B_0 meson in [17]. For some studies of mainly mass shifts for different hadrons in nuclear medium see for instance [18–46].

In the present study, we extend the works of [14,15] to investigate the modifications in the decay constants of the pseudoscalar B and D mesons in the framework of QCD sum rules. Considering contributions of various operators in nuclear medium, we calculate the appreciate forward scattering correlation function in hadronic and operator product expansion (OPE) sides in nuclear matter to obtain the QCD sum rules for the B–N and D–N scattering lengths and an extra phenomenological parameter entering the expressions of the modifications in the decay constants of the mesons under consideration. To study the electromagnetic structures and strong interactions of these mesons with other hadrons existing in the medium as well as for investigation of the B decays into the charmed D meson, we need to know also the modifications in the decay constants of these mesons due to nuclear medium besides the modifications in their mass. Our results can be useful in this respect. The results of the present work can also be used in analyses of the data obtained via heavy ion collisions held at different experiments.

The outline of the paper is as follows. In next section, after deriving the expressions of the modifications in the decay constants, we get the QCD sum rules for the B–N and D–N

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scattering lengths and an extra phenomenological parameter via calculating an appreciate forward scattering correlation function in terms of both the hadronic parameters and the QCD degrees of freedom in nuclear matter. Last section is devoted to the numerical analysis of the sum rules, obtaining the working regions for the auxiliary parameters entering the sum rules and numerical results on the shifts in the decay constants as well as the masses of the B and D mesons. We also compare the obtained results on the physical quantities under consideration with the existing predictions in the literature.

2 In-medium modifications of the decay constants of the *D* and *B* mesons via QCD sum rules

In order to calculate the shifts in the decay constants of D and B mesons in nuclear matter, we start with the following two-point correlation function which can be divided into the vacuum $\Pi_0(q)$ and the static one-nucleon $\Pi_N(q)$ parts in Fermi gas approximation for the nuclear matter. The $\Pi_N(q)$ function can also be approximated in the linear density of the nuclear matter as [14,47,48]

$$\Pi(q) = i \int \mathrm{d}^4x \mathrm{e}^{iq\cdot x} \langle \mathcal{T}[J_{B[D]}(x) J^{\dagger}_{B[D]}(0)]\rangle_{\rho_N} = \Pi_0(q) + \Pi_N(q) \simeq \Pi_0(q) + \frac{\rho_N}{2M_N} T_N(q), \quad (1)$$

where $\mathcal{T}$ is the time ordering operator, ρ_N is the density of the nuclear matter, M_N is the mass of the nucleon and $J_{B[D]}(x)$ denotes the interpolating current of the $B[D]$ meson. To find the shifts in the values of the decay constants, we shall consider the forward scattering amplitude $T_N(q)$ which can be written as

$$T_N(q_0 = \omega, \mathbf{q}) = i \int \mathrm{d}^4x \mathrm{e}^{iq\cdot x} \langle N(p)| \mathcal{T}[J_{B[D]}(x) J^{\dagger}_{B[D]}(0)] |N(p)\rangle, \quad (2)$$

where $q^\mu = (\omega, \mathbf{q})$ is the four-momentum of the meson and $|N(p)\rangle$ represents the isospin- and spin-averaged static nucleon state which is normalized covariantly as $\langle N(p)| N(p')\rangle = (2\pi)^3 2p_0 \delta^3(\mathbf{p}-\mathbf{p}')$ [14,17]. The pseudoscalar $B[D]$-meson interpolating field is taken as

$$J_{B[D]}(x) = \frac{\bar{u}(x) i\gamma_5 b[c](x) + \bar{b}[\bar{c}](x) i\gamma_5 u(x)}{2}, \quad (3)$$

where $u(x)$, $b(x)$, and $c(x)$ are quark fields. Note that in evaluating the $T_N(q)$ function we need to know the condensates $\langle \mathcal{O}_i\rangle_N$ which are related to the condensates $\langle \mathcal{O}_i\rangle_{\rho_N}$ via the following equation, valid at relatively low density [49]:

$$\langle \mathcal{O}_i\rangle_{\rho_N} = \langle \mathcal{O}_i\rangle_0 + \frac{\rho_N}{2M_N} \langle \mathcal{O}_i\rangle_N + o(\rho_N). \quad (4)$$

In the following, our main goal is to evaluate the forward scattering amplitude to find the shifts in the decay constants. According to the general philosophy of the method, we calculate this function via two different ways: in the phenomenological or hadronic side using the hadronic parameters and in the OPE or theoretical side in terms of QCD degrees of freedom. Equating these two representations of the same function, we obtain QCD sum rules for the shifts in the physical quantities under consideration. To suppress contributions of the higher states and continuum, a Borel transformation and continuum subtraction are applied to both sides of the obtained sum rules.

2.1 Hadronic side

The forward scattering amplitude $T_N(\omega, \mathbf{q})$ is calculated in terms of the hadronic parameters in the limit $\mathbf{q} \to 0$, around $\omega = m_{B[D]}$. Near the pole position of the pseudoscalar meson, $T_N(\omega, 0)$ is related to the $\mathbf{T}$-matrix for the forward $B[D]-N$ scattering amplitude [13]. The function $T_N(\omega, 0)$ is written as the following dispersion integrals [13]:

$$T_N(\omega, 0) = \int_{-\infty}^{+\infty} \mathrm{d}u \frac{\rho(u, \mathbf{q}=0)}{u - \omega - i\varepsilon} = \int_0^{\infty} \mathrm{d}u^2 \frac{\rho(u, \mathbf{q}=0)}{u^2 - \omega^2}, \quad (5)$$

where $\omega^2 \neq$positive real number and the spin-averaged spectral density $\rho(u, \mathbf{q}=0)$ can be expressed in terms of the spin-averaged $B[D]-N$ scattering $\mathbf{T}$-matrix, decay constant and mass of the $B[D]$ meson as well as phenomenological parameters a, b, and c in the following way:

$$\begin{aligned}
\rho(u > 0, \mathbf{q}=0) &= -\frac{f^2_{B[D]} m^4_{B[D]}}{\pi m^2_{b[c]}} \mathbf{Im}\left[\frac{\mathbf{T}_{B[D]N}(u,0)}{(u^2 - m^2_{B[D]} + i\varepsilon)^2}\right] + \cdots \\
&= -\frac{f^2_{B[D]} m^4_{B[D]}}{\pi m^2_{b[c]}} \left\{\mathbf{Im} \frac{1}{(u^2 - m^2_{B[D]} + i\varepsilon)^2} \mathbf{Re}[\mathbf{T}_{B[D]N}(u,0)] + \mathbf{Re} \frac{1}{(u^2 - m^2_{B[D]} + i\varepsilon)^2} \mathbf{Im}[\mathbf{T}_{B[D]N}(u,0)]\right\} + \cdots \quad (6) \\
&\equiv a \frac{d}{du^2} \delta(u^2 - m^2_{B[D]}) + b\, \delta(u^2 - m^2_{B[D]}) + c\, \delta(u^2 - s_0), \quad (7)
\end{aligned}$$

where . . . in Eq. (6) denotes the contribution of higher states and continuum which is not associated with the $B[D]-N$ scattering. It is equivalent to the third term in Eq. (7) which represents the scattering contribution in the continuum part of the $B[D]$ current starting at the threshold s_0. Applying the Borel transformation and continuum subtraction suppresses this contribution. Note that the first term proportional to the parameter a in Eq. (7) denotes the double-pole term and corresponds to the on-shell effect of the $\mathbf{T}$-matrix. The second term proportional to the parameter b in Eq. (7) denotes the single-pole term and corresponds to the off-shell effect of the $\mathbf{T}$-matrix. The phenomenological parameters a and b are found as

$$a = -\frac{f_{B[D]}^2 m_{B[D]}^4}{m_{b[c]}^2}\mathbf{Re}[\mathbf{T}_{B[D]N}(u,0)]|_{u=m_{B[D]}}$$
$$= -\frac{8\pi f_{B[D]}^2 m_{B[D]}^4 (M_N + m_{B[D]})}{m_{b[c]}^2} a_{B[D]},$$
$$b = -\frac{f_{B[D]}^2 m_{B[D]}^4}{m_{b[c]}^2}\frac{d}{du^2}\mathbf{Re}[\mathbf{T}_{B[D]N}(u,0)]|_{u=m_{B[D]}}, \tag{8}$$

where the parameter $a_{B[D]}$ is the $B[D]-N$ scattering length [13]. The decay constant $f_{B[D]}$ of the pseudoscalar $B[D]$ meson is defined as

$$\langle 0|J_{B[D]}(0)|B[D]\rangle = \frac{f_{B[D]} m_{B[D]}^2}{m_{b[c]}}. \tag{9}$$

Combining Eqs. (7), (5), (2), and (1), we can relate the phenomenological parameters a and b extracted from the forward scattering amplitude T_N with the shifts in the mass and decay constant of the $B[D]$ meson as [13]

$$\Pi^{\mathrm{HAD}}(\omega,0) \propto \frac{F}{m_{B[D]}^2-\omega^2} + \frac{\rho_N}{2M_N} \times \left\{\frac{a}{(m_{B[D]}^2-\omega^2)^2} + \frac{b}{m_{B[D]}^2-\omega^2}\right\}$$
$$\simeq \frac{F+\delta F}{(m_{B[D]}^2+\Delta m_{B[D]}^2)-\omega^2} \tag{10}$$

where

$$F = \frac{f_{B[D]}^2 m_{B[D]}^4}{m_{b[c]}^2},$$
$$\delta F = \frac{\rho_N}{2M_N} b,$$
$$\Delta m_{B[D]}^2 = -\frac{\rho_N}{2M_N F} a. \tag{11}$$

Using the modified mass in nuclear matter, $m_{B[D]}^* = m_{B[D]} + \delta m_{B[D]} = \sqrt{m_{B[D]}^2 + \Delta m_{B[D]}^2}$, the mass shift of B[D] meson is obtained as:

$$\delta m_{B[D]} = 2\pi \frac{M_N + m_{B[D]}}{M_N m_{B[D]}} \rho_N a_{B[D]}. \tag{12}$$

From Eq. (11), the shift in decay constant of the $B[D]$ meson is also obtained as

$$\delta f_{B[D]} = \frac{m_{b[c]}^2}{2 f_{B[D]} m_{B[D]}^4}\left(\frac{\rho_N}{2M_N} b - \frac{4 f_{B[D]}^2 m_{B[D]}^3}{m_{b[c]}^2}\delta m_{B[D]}\right). \tag{13}$$

As is clear from the above relations, to find the shifts in the mass and decay constant, we need to calculate the phenomenological parameters a and b using the forward scattering amplitude calculated both in hadronic and OPE sides.

In the low energy limit $\omega \to 0$, the $T_N^{\mathrm{HAD}}(\omega,0)$ is equivalent to the Born term $T_N^{\mathrm{Born}}(\omega,0)$. Hence, the forward scattering amplitude on the hadronic side can be written as

$$T_N^{\mathrm{HAD}}(\omega,0) = T_N^{\mathrm{Born}}(\omega,0) + \frac{a}{(m_{B[D]}^2-\omega^2)^2} + \frac{b}{m_{B[D]}^2-\omega^2} + \frac{c}{s_0^2-\omega^2}, \tag{14}$$

with the condition

$$\frac{a}{m_{B[D]}^4} + \frac{b}{m_{B[D]}^2} + \frac{c}{s_0} = 0. \tag{15}$$

The Born term can be determined by the Born diagrams at the tree level [13,14]. To calculate it, we consider the contributions of the baryons $\Lambda_{b[c]}$ and $\Sigma_{b[c]}$ in the medium produced by the interaction of B[D] with the nucleon, i.e.

$$B^-(b\bar{u}) + p(uud) \ or \ n(udd) \to \Lambda_b^0(udb) \ or \ \Sigma_b^-(ddb),$$
$$D^0(c\bar{u}) + p(uud) \ or \ n(udd) \to \Lambda_c^+, \Sigma_c^+(udc) \ or \ \Sigma_c^0(ddc) \tag{16}$$

The Born term $T_N^{\mathrm{Born}}(\omega,0)$ is obtained as [14]

$$T^{\mathrm{Born}}(\omega,0) = \frac{2M_N(M_N+M_{\mathcal{B}})m_{B[D]}^4 f_{B[D]}^2}{[\omega^2-(M_N+M_{\mathcal{B}})^2](\omega^2-m_{B[D]}^2)^2(m_u+m_{b[c]})^2} g_{NB[D]\mathcal{B}}^2(\omega^2). \tag{17}$$

where $\mathcal{B}$ denotes the $\Lambda_{b[c]}$ or $\Sigma_{b[c]}$ baryon and $g_{NB[D]\mathcal{B}}(\omega^2)$ is the strong coupling constant among the $B[D]$ meson, nucleon and $\mathcal{B}$ baryon.

After the Borel transformation and using the quark–hadron duality assumption, the hadronic side of the current-nucleon forward scattering amplitude is obtained as (see also [14]):

$$\widehat{\mathbf{B}}T_N^{\mathrm{HAD}} = a\left(\frac{1}{M^2}e^{-m_{B[D]}^2/M^2} - \frac{s_0}{m_{B[D]}^4}e^{-s_0/M^2}\right) + b\left(e^{-m_{B[D]}^2/M^2} - \frac{s_0}{m_{B[D]}^2}e^{-s_0/M^2}\right)$$
$$+\frac{2f_{B[D]}^2 m_{B[D]}^4 M_N(M_N+M_{\mathcal{B}})}{[(M_N+M_{\mathcal{B}})^2-m_{B[D]}^2](m_u+m_{b[c]})^2} g_{NB[D]\mathcal{B}}^2 \times\left[-\frac{e^{-(M_N+M_{\mathcal{B}})^2/M^2}}{(M_N+M_{\mathcal{B}})^2-m_{B[D]}^2} + \left(\frac{1}{(M_N+M_{\mathcal{B}})^2-m_{B[D]}^2} - \frac{1}{M^2}\right)e^{-m_{B[D]}^2/M^2}\right]. \tag{18}$$

2.2 OPE side

The OPE side of the forward scattering amplitude is obtained via inserting the explicit form of the interpolating current $J_{B[D]}$ into Eq. (2). After contracting out all quark pairs via Wick's theorem, we get

$$T_N^{\rm OPE} = \frac{i}{4}\int d^4x e^{iq.x}\left\langle N(p)\left|Tr\left[S_Q(-x)\gamma_5 S_u(x)\gamma_5 + S_u(-x)\gamma_5 S_Q(x)\gamma_5\right]\right|N(p)\right\rangle, \quad (19)$$

where S_u is light quark and S_Q with $Q = b$ or c is the heavy quark propagator. The light quark propagator in the fixed-point gauge at nuclear medium is given by [20,50]

$$\begin{aligned} S_u^{ij}(x) &\equiv \langle N(p)|T[q^i(x)\bar{q}^j(0)]|N(p)\rangle \\ &= \frac{i}{2\pi^2}\delta^{ij}\frac{1}{(x^2)^2}\not{x} - \frac{m_q}{4\pi^2}\delta^{ij}\frac{1}{x^2} + \chi_q^i(x)\bar{\chi}_q^j(0) \\ &\quad - \frac{ig_s}{32\pi^2}F_{\mu\nu}^A(0)t^{ij,A}\frac{1}{x^2}[\not{x}\sigma^{\mu\nu} + \sigma^{\mu\nu}\not{x}] + \cdots, \end{aligned} \quad (20)$$

where χ_q^i and $\bar{\chi}_q^j$ are the Grassmann background quark fields and $F_{\mu\nu}^A$ is classical background gluon field. The first and second terms in the above equation stand for free or perturbative part, and the third and fourth terms denote the non-perturbative part or contributions due to the background quark and gluon fields. The heavy quark propagator is taken as

$$\begin{aligned} S_Q^{ij}(x) &\equiv \langle N(p)|T[Q^i(x)\bar{Q}^j(0)]|N(p)\rangle \\ &= \frac{i}{(2\pi)^4}\int d^4k e^{(-ik\cdot x)}\left\{\frac{\delta^{ij}}{\not{k} - m_Q}\right. \\ &\quad - \frac{g_s G_{\alpha\beta}^n t_{ij}^n}{4}\frac{\sigma^{\alpha\beta}(\not{k}+m_Q) + (\not{k}+m_Q)\sigma^{\alpha\beta}}{(k^2-m_Q^2)^2} \\ &\quad \left. + \frac{\delta_{ij}\langle g_s^2 GG\rangle}{12}\frac{m_Q k^2 + m_Q^2\not{k}}{(k^2-m_Q^2)^4} + \cdots\right\}. \end{aligned} \quad (21)$$

The next step is to use the light and heavy quark propagators in Eq. (19). As we deal only with the shifts in the mass and decay constant compared to their vacuum values, it is enough to consider only the terms having non-perturbative effects. To go further, we need to define the products of the Grassmann background quark fields and classical background gluon fields in terms of the ground-state matrix elements of the corresponding quark and gluon operators at nuclear medium [20],

$$\begin{aligned} &\chi_{i\alpha}^q(x)\bar{\chi}_{j\beta}^q(0) = \langle q_{i\alpha}(x)\bar{q}_{j\beta}(0)\rangle_N, \quad F_{\kappa\lambda}^A F_{\mu\nu}^B = \langle G_{\kappa\lambda}^A G_{\mu\nu}^B\rangle_N, \\ &\chi_{i\alpha}^q\bar{\chi}_{j\beta}^q F_{\mu\nu}^A = \langle q_{i\alpha}\bar{q}_{j\beta}G_{\mu\nu}^A\rangle_N, \quad \chi_{i\alpha}^q\bar{\chi}_{j\beta}^q\chi_{k\gamma}^q\bar{\chi}_{l\delta}^q = \langle q_{i\alpha}\bar{q}_{j\beta}q_{k\gamma}\bar{q}_{l\delta}\rangle_N. \end{aligned} \quad (22)$$

The matrix elements $\langle q_{i\alpha}(x)\bar{q}_{j\beta}(0)\rangle_N$ and $\langle g_s q_{i\alpha}\bar{q}_{j\beta}G_{\mu\nu}^A\rangle_N$ are defined as [20]

$$\begin{aligned} \langle q_{i\alpha}(x)\bar{q}_{j\beta}(0)\rangle_N &= -\frac{\delta_{ij}}{12}\left[\left(\langle\bar{q}q\rangle_N + x^\mu\langle\bar{q}D_\mu q\rangle_N\right.\right. \\ &\quad \left. + \frac{1}{2}x^\mu x^\nu\langle\bar{q}D_\mu D_\nu q\rangle_N + \cdots\right)\delta_{\alpha\beta} + \left(\langle\bar{q}\gamma_\lambda q\rangle_N\right. \\ &\quad \left.\left. + x^\mu\langle\bar{q}\gamma_\lambda D_\mu q\rangle_N + \frac{1}{2}x^\mu x^\nu\langle\bar{q}\gamma_\lambda D_\mu D_\nu q\rangle_N + \cdots\right)\gamma_{\alpha\beta}^\lambda\right], \end{aligned} \quad (23)$$

and

$$\begin{aligned} &\langle g_s q_{i\alpha}\bar{q}_{j\beta}G_{\mu\nu}^A\rangle_N \\ &= -\frac{t_{ij}^A}{96}\left\{\langle g_s\bar{q}\sigma\cdot Gq\rangle_N\left[\sigma_{\mu\nu} + i(u_\mu\gamma_\nu - u_\nu\gamma_\mu)\not{u}\right]_{\alpha\beta}\right. \\ &\quad + \langle g_s\bar{q}\not{u}\sigma\cdot Gq\rangle_N\left[\sigma_{\mu\nu}\not{u} + i(u_\mu\gamma_\nu - u_\nu\gamma_\mu)\right]_{\alpha\beta} \\ &\quad - 4\left(\langle\bar{q}u\cdot Du\cdot Dq\rangle_N + im_q\langle\bar{q}\not{u}u\cdot Dq\rangle_N\right) \\ &\quad \left.\times\left[\sigma_{\mu\nu} + 2i(u_\mu\gamma_\nu - u_\nu\gamma_\mu)\not{u}\right]_{\alpha\beta}\right\}, \end{aligned} \quad (24)$$

where $D_\mu = \frac{1}{2}(\gamma_\mu \not{D} + \not{D}\gamma_\mu)$. The matrix element of the four-dimension gluon condensate is also defined as

$$\begin{aligned} &\langle G_{\kappa\lambda}^A G_{\mu\nu}^B\rangle_N \\ &= \frac{\delta^{AB}}{96}\left[\langle G^2\rangle_N(g_{\kappa\mu}g_{\lambda\nu} - g_{\kappa\nu}g_{\lambda\mu}) + O(\langle\mathbf{E}^2 + \mathbf{B}^2\rangle_N)\right], \end{aligned} \quad (25)$$

where we neglect the last term in this equation because of its small contribution. We also ignore from the four-quark condensate contributions in Eq. (22).

Various condensates appear in calculations are defined in terms of the four-velocity u^μ of the nuclear medium as [11, 20]

$$\langle\bar{q}\gamma_\mu q\rangle_N = \langle\bar{q}\not{u}q\rangle_N u_\mu, \quad (26)$$

$$\langle\bar{q}D_\mu q\rangle_N = \langle\bar{q}u\cdot Dq\rangle_N u_\mu = -im_q\langle\bar{q}\not{u}q\rangle_N u_\mu, \quad (27)$$

$$\begin{aligned} \langle\bar{q}\gamma_\mu D_\nu q\rangle_N &= \frac{4}{3}\langle\bar{q}\not{u}u\cdot Dq\rangle_N\left(u_\mu u_\nu - \frac{1}{4}g_{\mu\nu}\right) \\ &\quad + \frac{i}{3}m_q\langle\bar{q}q\rangle_N(u_\mu u_\nu - g_{\mu\nu}), \end{aligned} \quad (28)$$

$$\begin{aligned} \langle\bar{q}D_\mu D_\nu q\rangle_N &= \frac{4}{3}\langle\bar{q}u\cdot Du\cdot Dq\rangle_N\left(u_\mu u_\nu - \frac{1}{4}g_{\mu\nu}\right) \\ &\quad - \frac{1}{6}\langle g_s\bar{q}\sigma\cdot Gq\rangle_N(u_\mu u_\nu - g_{\mu\nu}), \end{aligned} \quad (29)$$

$$\begin{aligned} \langle\bar{q}\gamma_\lambda D_\mu D_\nu q\rangle_N &= 2\langle\bar{q}\not{u}u\cdot Du\cdot Dq\rangle_N\left[u_\lambda u_\mu u_\nu\right. \\ &\quad \left. - \frac{1}{6}(u_\lambda g_{\mu\nu} + u_\mu g_{\lambda\nu} + u_\nu g_{\lambda\mu})\right] \\ &\quad - \frac{1}{6}\langle g_s\bar{q}\not{u}\sigma\cdot Gq\rangle_N(u_\lambda u_\mu u_\nu - u_\lambda g_{\mu\nu}), \end{aligned} \quad (30)$$

where the equation of motion have been used and terms $O(m_q^2)$ have been neglected due to their very small contributions [20].

Making use of all above equations, the OPE side of the T_N function in the rest frame of the nuclear matter in the Borel scheme is obtained as

$$\begin{aligned}
&\widehat{\mathbf{B}}T_N^{\mathrm{OPE}} \\
&= \frac{1}{3}\frac{\mathrm{e}^{-m_Q^2/M^2}}{M^4}\left\{-m_Q\left(-2m_Q^2+M^2+2p_0^2\right)\langle\bar{q}g_s\sigma Gq\rangle_N\right. \\
&\quad -4m_Q\left(m_Q^2-2M^2+4p_0^2\right)\langle\bar{q}D_0D_0q\rangle_N \\
&\quad +4M^2\left(-m_Q^2+M^2+4p_0^2\right)\langle q^\dagger iD_0q\rangle_N \\
&\quad \left.+2M^2\left[2m_Q^2m_u-3m_QM^2+m_u\left(M^2-2p_0^2\right)\right]\langle\bar{q}q\rangle_N\right\} \\
&\quad +\frac{1}{12\pi^2}\langle g_s^2G^2\rangle_N\int_0^\infty \mathrm{d}\alpha\,\frac{\mathrm{e}^{m_Q^2/(4\alpha-M^2)}m_Q}{(4\alpha-M^2)^4} \\
&\quad \times\left\{16\alpha^2(m_Q+3m_u)+M^2\left(-m_Q^2m_u+3m_QM^2+3m_uM^2\right)\right. \\
&\quad \left.-4\alpha\left(m_Q^3-m_Q^2m_u+4m_QM^2+6m_uM^2\right)\right\} \\
&\quad \times\theta\left[\frac{1}{-4\alpha+M^2}\right]-\frac{m_Q\mathrm{e}^{-m_Q^2/M^2}}{M^2}\langle\bar{q}g_s\sigma Gq\rangle_N. \qquad (31)
\end{aligned}$$

2.3 QCD sum rules for the phenomenological parameters a and b

In this subsection, the Borel transformed hadronic and OPE sides of the T_N function are equated to find the QCD sum rules for the parameters a and b, i.e.,

$$\widehat{\mathbf{B}}T_N^{\mathrm{HAD}}=\widehat{\mathbf{B}}T_N^{\mathrm{OPE}}. \qquad (32)$$

As we have two unknowns, we need one more equation which is found applying the derivative with respect to $\frac{1}{M^2}$ to both sides of Eq. (32):

$$\frac{\partial}{\partial(1/M^2)}\left\{\widehat{\mathbf{B}}T_N^{\mathrm{HAD}}\right\}=\frac{\partial}{\partial(1/M^2)}\left\{\widehat{\mathbf{B}}T_N^{\mathrm{OPE}}\right\}. \qquad (33)$$

By simultaneous solving of Eqs. (32) and (33), we obtain the following sum rules for the parameters a and b:

$$\begin{aligned}
a &= \frac{f_{B[D]}^2g_{NB[D]\mathcal{B}}^2}{\Delta\,\Delta'(m_u+m_{b[c]})^2}\left\{-2m_{B[D]}^4M_N(M_N+M_{\mathcal{B}})\right. \\
&\quad \times\exp\left[-\frac{m_{B[D]}^2+(M_{\mathcal{B}}+M_N)^2}{M^2}\right] \\
&\quad +\frac{2s_0m_{B[D]}^2M_N(M_N+M_{\mathcal{B}})\left((M_N+M_{\mathcal{B}})^2-s_0\right)}{\Delta'} \\
&\quad \times\exp\left[-\frac{(M_{\mathcal{B}}+M_N)^2+s_0}{M^2}\right] \\
&\quad +2m_{B[D]}^4M_N(M_N+M_{\mathcal{B}})\exp\left[-\frac{2m_{B[D]}^2}{M^2}\right] \\
&\quad -\frac{2s_0m_{B[D]}^2M_N(M_N+M_{\mathcal{B}})}{\Delta'M^2}\left[m_{B[D]}^4+M_N^2M^2\right. \\
&\quad +M_N^2s_0-M^2s_0-m_{B[D]}^2(M_N^2+s_0) \\
&\quad \left.+(s_0+M^2-m_{B[D]}^2)(M_{\mathcal{B}}^2+2M_NM_{\mathcal{B}})\right] \\
&\quad \left.\times\exp\left[-\frac{m_{B[D]}^2+s_0}{M^2}\right]\right\}+\frac{1}{\Delta m_{B[D]}^2} \\
&\quad \times\left\{m_{B[D]}^2\left[m_{B[D]}^2\widehat{\mathbf{B}}T_N^{\mathrm{OPE}}+\left(\frac{\mathrm{d}}{\mathrm{d}\frac{1}{M^2}}\widehat{\mathbf{B}}T_N^{\mathrm{OPE}}\right)\right]\right. \\
&\quad \times\exp\left[-\frac{m_{B[D]}^2}{M^2}\right]-s_0\left[s_0\widehat{\mathbf{B}}T_N^{\mathrm{OPE}}+\left(\frac{\mathrm{d}}{\mathrm{d}\frac{1}{M^2}}\widehat{\mathbf{B}}T_N^{\mathrm{OPE}}\right)\right] \\
&\quad \left.\times\exp\left[-\frac{s_0}{M^2}\right]\right\}, \qquad (34)
\end{aligned}$$

$$\begin{aligned}
b &= \frac{f_{B[D]}^2g_{NB[D]\mathcal{B}}^2}{\Delta'\Delta\Phi}\left\{-\frac{2M_Ns_0(M_N+M_{\mathcal{B}})}{\Delta'M^2(m_u+m_{b[c]})^2}\right. \\
&\quad \times\left[M_N^2M^2+M_{\mathcal{B}}^2(m_{B[D]}^2+M^2)\right. \\
&\quad \left.+2M_{\mathcal{B}}M_N(m_{B[D]}^2+M^2)+m_{B[D]}^2(M_N^2-M^2-s_0)\right] \\
&\quad \times\exp\left[-\frac{m_{B[D]}^2+\left((M_N+M_{\mathcal{B}})^2+s_0\right)}{M^2}\right] \\
&\quad +\frac{2m_{B[D]}^4M_N(M_N+M_{\mathcal{B}})}{M^2(m_u+m_{b[c]})^2} \\
&\quad \times\exp\left[-\frac{2m_{B[D]}^2+(M_N+M_{\mathcal{B}})^2}{M^2}\right] \\
&\quad +\frac{2s_0^2M_N(M_N+M_{\mathcal{B}})(M_{\mathcal{B}}^2+2M_{\mathcal{B}}M_N+M_N^2-s_0)}{\Delta'm_{B[D]}^2(m_u+m_{b[c]})^2} \\
&\quad \times\exp\left[-\frac{(M_N+M_{\mathcal{B}})^2+2s_0}{M^2}\right] \\
&\quad +\frac{2s_0m_{B[D]}^2M_N(M_N+M_{\mathcal{B}})(m_{B[D]}^2-s_0)}{\Delta'M^2(m_u+m_{b[c]})^2} \\
&\quad \times\exp\left[-\frac{2m_{B[D]}^2+s_0}{M^2}\right] \\
&\quad -\frac{2s_0^2M_N(M_N+M_{\mathcal{B}})(m_{B[D]}^2-s_0)}{\Delta'm_{B[D]}^2(m_u+m_{b[c]})^2} \\
&\quad \times\exp\left[-\frac{m_{B[D]}^2+2s_0}{M^2}\right]-\frac{2m_{B[D]}^4M_N(M_N+M_{\mathcal{B}})}{M^2(m_u+m_{b[c]})^2}
\end{aligned}$$

$$\times \exp\left[-\frac{3m_{B[D]}^2}{M^2}\right]$$
$$+\frac{2s_0 M_N(M_N+M_{\mathcal{B}})\left(M^4+m_{B[D]}^2(M^2-m_{B[D]}^2+s_0)\right)}{M^4(m_u+m_{b[c]})^2}$$
$$\times \exp\left[-\frac{2m_{B[D]}^2+s_0}{M^2}\right]$$
$$+\frac{2s_0^2 M_N(M_N+M_{\mathcal{B}})(m_{B[D]}^2-M^2-s_0)}{M^2 m_{B[D]}^2(m_u+m_{b[c]})^2}$$
$$\times \exp\left[-\frac{m_{B[D]}^2+2s_0}{M^2}\right]$$
$$+\frac{2\Delta m_{B[D]}^4 M_N(M_N+M_{\mathcal{B}})(\Delta'-M^2)}{\Delta' M^2(m_u+m_{b[c]})^2}$$
$$\times \exp\left[-\frac{m_{B[D]}^2}{M^2}\right]+\frac{2\Delta m_{B[D]}^4 M_N(M_N+M_{\mathcal{B}})}{\Delta'(m_u+m_{b[c]})^2}$$
$$\times \exp\left[-\frac{(M_N+M_{\mathcal{B}})^2}{M^2}\right]\Bigg\}+\frac{\widehat{\mathbf{B}}T_N^{\mathrm{OPE}}}{\Phi\Delta}\Bigg\{\Delta-\frac{m_{B[D]}^2}{M^2}$$
$$\times \exp\left[-\frac{2m_{B[D]}^2}{M^2}\right]+\frac{s_0(M^2+s_0)}{M^2 m_{B[D]}^2}$$
$$\times \exp\left[-\frac{m_{B[D]}^2+s_0}{M^2}\right]-\frac{s_0^3}{m_{B[D]}^6}\exp\left[-\frac{2s_0}{M^2}\right]\Bigg\}$$
$$+\frac{1}{\Delta\Phi}\left(\frac{\mathrm{d}}{\mathrm{d}\frac{1}{M^2}}\widehat{\mathbf{B}}T_N^{\mathrm{OPE}}\right)\Bigg\{\frac{s_0(M^2+m_{B[D]}^2)}{M^2 m_{B[D]}^4}$$
$$\times \exp\left[-\frac{m_{B[D]}^2+s_0}{M^2}\right]-\frac{1}{M^2}\exp\left[-\frac{2m_{B[D]}^2}{M^2}\right]$$
$$-\frac{s_0^2}{m_{B[D]}^6}\exp\left[-\frac{2s_0}{M^2}\right]\Bigg\}, \tag{35}$$

where

$$\Delta'=(M_{\mathcal{B}}+M_N)^2-m_{B[D]}^2$$
$$\Delta=\exp\left[-\frac{2m_{B[D]}^2}{M^2}\right]-\frac{2s_0}{m_{B[D]}^2}$$
$$\times \exp\left[-\frac{m_{B[D]}^2+s_0}{M^2}\right]+\frac{s_0}{M^2}\exp\left[-\frac{m_{B[D]}^2+s_0}{M^2}\right]$$
$$+\frac{s_0^2}{m_{B[D]}^4}\exp\left[-\frac{m_{B[D]}^2+s_0}{M^2}\right]-\frac{s_0^2}{m_{B[D]}^2 M^2}$$
$$\times \exp\left[-\frac{m_{B[D]}^2+s_0}{M^2}\right]$$
$$\Phi=\exp\left[-\frac{m_{B[D]}^2}{M^2}\right]-\frac{s_0}{m_{B[D]}^2}\exp\left[-\frac{s_0}{M^2}\right]. \tag{36}$$

3 Numerical results and discussion

In order to numerically analyze the QCD sum rules obtained in the previous section, we need to know the numerical values of the $\langle\mathcal{O}_i\rangle_N$ condensates. As we deal only with the shifts in the physical quantities under consideration with respect to their vacuum values, we set the vacuum condensates $\langle\mathcal{O}_i\rangle_0$ to zero in Eq. (4) and find the values of the $\langle\mathcal{O}_i\rangle_N$ condensates in terms of the condensates $\langle\mathcal{O}_i\rangle_{\rho_N}$, i.e. $\langle\mathcal{O}_i\rangle_N=\frac{2M_N}{\rho_N}\langle\mathcal{O}_i\rangle_{\rho_N}$. Using this relation and the values of condensates $\langle\mathcal{O}_i\rangle_{\rho_N}$ presented in [11,12,19,20] we find the values of the condensates $\langle\mathcal{O}_i\rangle_N$ as depicted in Table 1 (see also [17]). To proceed with the numerical analysis, we also need the values of some other input parameters, like the quark masses, which are also presented in Table 1. Note that, in the present study, we use the quark masses in the $\overline{MS}$ scheme.

Besides these input parameters, the sum rules for the parameters a and b contain two auxiliary objects, namely the Borel mass parameter M^2 and continuum threshold s_0. According to the general philosophy of the method used, the physical quantities should be independent of these auxiliary objects. Hence, we should look for "working regions" of these parameters such that at these regions, the physical quantities have weak dependence on M^2 and s_0. Our numerical calculations show that in the intervals 25 GeV$^2\le M^2\le$ 40 GeV^2 and 4 GeV$^2\le M^2\le 8$ GeV^2, respectively, in the

Table 1 Numerical values for input parameters [11,12,14,17,19,20,51,52]. ρ_N^{sat} means the saturation nuclear matter density

Parameters	Values
$p_0^{B[D]}$	5.279 [1.870] GeV
$m_{b[c]}$	4.18 [1.275] GeV
M_N	0.938 GeV
$M_{\mathcal{B}_{b[c]}}$	5.619 [2.4] GeV
$m_{B[D]}$	5.279 [1.870] GeV
$g_{NB[D]\mathcal{B}}$	6.74 GeV
$f_{B[D]}$	0.17 [0.2067] GeV
ρ_N^{sat}	$(0.11)^3$ GeV^3
$\langle q^\dagger q\rangle_N$	$\frac{3}{2}(2M_N)$
m_q	6 MeV
σ_N	0.045 GeV
$\langle\bar{q}q\rangle_N$	$\frac{\sigma_N}{2m_q}(2M_N)$
$\langle q^\dagger g_s\sigma Gq\rangle_N$	-0.33 GeV$^2(2M_N)$
$\langle q^\dagger iD_0q\rangle_N$	0.18 GeV$(2M_N)$
$\langle\bar{q}iD_0q\rangle_N$	$\simeq 0$
$\langle\bar{q}g_s\sigma Gq\rangle_N$	3 GeV$^2(2M_N)$
$\langle\bar{q}iD_0iD_0q\rangle_N$	$0.3\ GeV^2(2M_N)-\frac{1}{8}\langle\bar{q}g_s\sigma Gq\rangle_N$
$\langle q^\dagger iD_0iD_0q\rangle_N$	0.031 GeV$^2(2M_N)-\frac{1}{12}\langle q^\dagger g_s\sigma Gq\rangle_N$
$\langle\frac{\alpha_s}{\pi}G^2\rangle_N$	-0.65 GeV$(2M_N)$

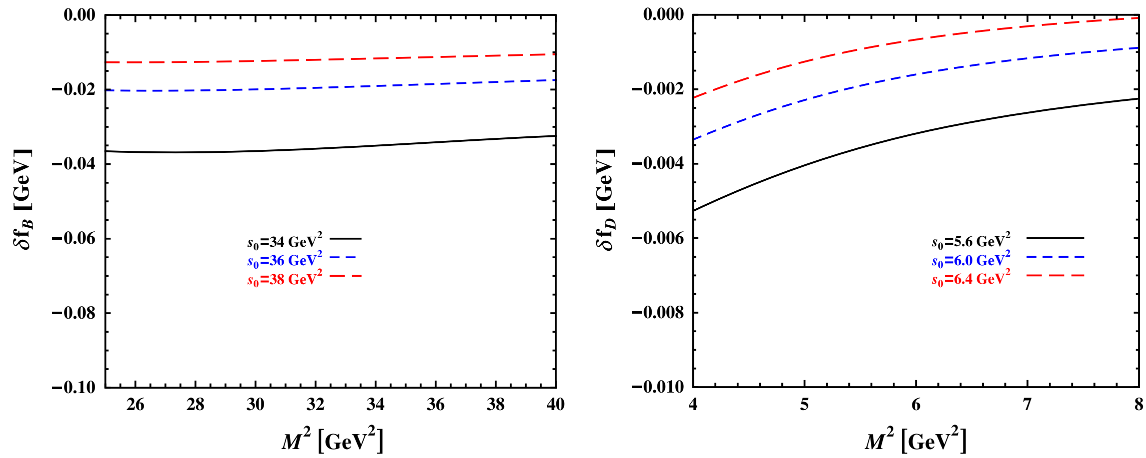

Fig. 1 The shift of B meson's decay constant in nuclear matter versus the Borel mass M^2 at three different values of continuum threshold (*left panel*). The same, but for shift in decay constant of the D meson (*right panel*)

B and D channels, the dependence of the shifts in the physical quantities are weak. Also we see that in the intervals 34 GeV$^2 \leq s_0 \leq 38$ GeV^2 and 5.6 GeV$^2 \leq s_0 \leq 6.4$ GeV^2, respectively for the B and D mesons, the results demonstrate a weak dependence on the continuum threshold.

Making use of all input parameters and working regions for the auxiliary parameters we depict the dependence of the shifts in the decay constants of the B and D mesons on the Borel mass parameter at different fixed values of the continuum threshold in Fig. 1. The left panel in this figure belongs to the shift of the decay constant of the B meson, while the right panel includes the variations of this quantity with respect to M^2 in D channel. By a quick glance at this figure, we see the following.

- The decay constant at B channel shows a good stability with respect to the variations of M^2 in its working region, while this quantity weakly depend on M^2 at D channel. The absolute value of the shift in the decay constant of the D meson decreases by increasing the value of M^2, such that at the upper band of the Borel mass parameter and a higher value of the continuum threshold this shift becomes very small.
- The shifts of decay constants due to nuclear medium are negative in both B and D channels.
- The shift in the decay constant of B meson is roughly 10 times bigger than that of the D meson.
- Increasing the value of the continuum threshold in both channels ends up with decrease in the absolute values of the shifts in decay constants.

Extracted from Fig. 1, we depict the average values of the shifts in the decay constants of B and D mesons in Table 2. The quoted errors in the values of the shifts in decay constants belong to the uncertainties coming from both the determination of the working region for auxiliary parameters and the errors of other input parameters.

Table 2 Average values of the shifts in the decay constants of the B and D mesons

	δf_B (GeV)	δf_D (GeV)
Present work	-0.023 ± 0.007	-0.002 ± 0.001

In order to make a comparison of the results on the mass shifts with the previous theoretical predictions, we also numerically analyze these shifts in the B and D channels. For this aim, we depict the dependence of the mass shifts on M^2 at different fixed values of s_0 in Fig. 2. From this figure, we conclude that

- The shifts in the masses of both B and D mesons are negative.
- The shift in the mass of the B meson is roughly 5 times greater than that of the D meson.
- The shifts in both B and D channels demonstrate good stabilities with respect to the variations of the Borel mass parameter.

From Fig. 2, we also extract the values of the shifts in the masses of the mesons under consideration as presented in Table 3. For comparison, we also depict the predictions of some previous theoretical works in the same table. From this table, we see that our result on the mass shift in D channel is in a good consistency with the result of [14] which uses the same method and interpolating current. However, the prediction of [15] in this channel is in opposite sign with ours and prediction of [14], although it predicts the same value in magnitude. As far as the shift in the mass of B channel is considered, the only existing prediction belongs to [15]

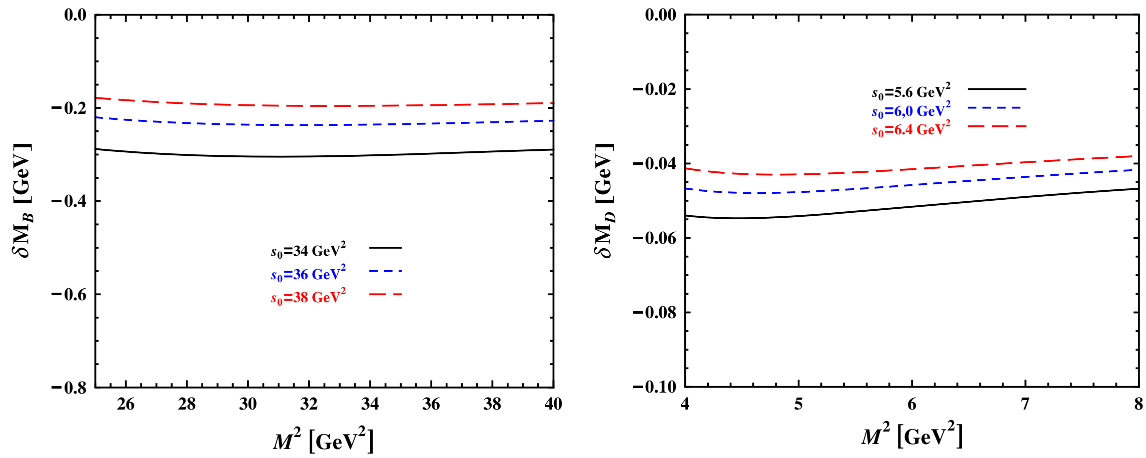

Fig. 2 The B meson mass shift in nuclear matter versus the Borel mass M^2 at three different values of continuum threshold (*left panel*). The same, but for D meson mass shift (*right panel*)

Table 3 Average values of the shifts in the masses of the D and B mesons

	δm_B (GeV)	δm_D (GeV)
Present work	-0.242 ± 0.062	-0.046 ± 0.007
[14]	–	-0.048 ± 0.008
[15]	~ 0.060	~ 0.045

which is different from our result in both sign and magnitude. Note that in [15] the authors use the interpolating currents $J_{D^+} = i\bar{d}\gamma_5 c$ and $J_{B^+} = i\bar{b}\gamma_5 u$ or $J_{B^0} = i\bar{b}\gamma_5 d$ in D and B channels, respectively.

At the end of this section, we would like to discuss the dependence of the results on the shifts in the decay constants and masses to the nuclear matter density. In the above numerical results, we have used the value of saturation density, i.e. $\rho_N^{\text{sat}} = (0.11)^3$ GeV^3. In order to see how the results depend on the nuclear matter density, we plot the shifts in the decay constants and masses versus $\rho_N/\rho_N^{\text{sat}}$ in Figs. 3 and 4 at the average values of the Borel mass parameter and continuum threshold.

As also expected from Eqs. (12) and (13), these figures show that the shifts in the physical quantities under consideration linearly depend on the nuclear matter density. The absolute values of the shifts in the decay constants and masses increase by increasing the nuclear matter density.

In summary, we calculated the shifts in the decay constants and masses of the pseudoscalar D and B mesons due to nuclear matter in the framework of the QCD sum rules. We found considerable negative shifts in the values of the considered quantities except for the shift in the decay constant of the D meson which is very small. We compared our results on the mass shifts in D and B channels with the pre-

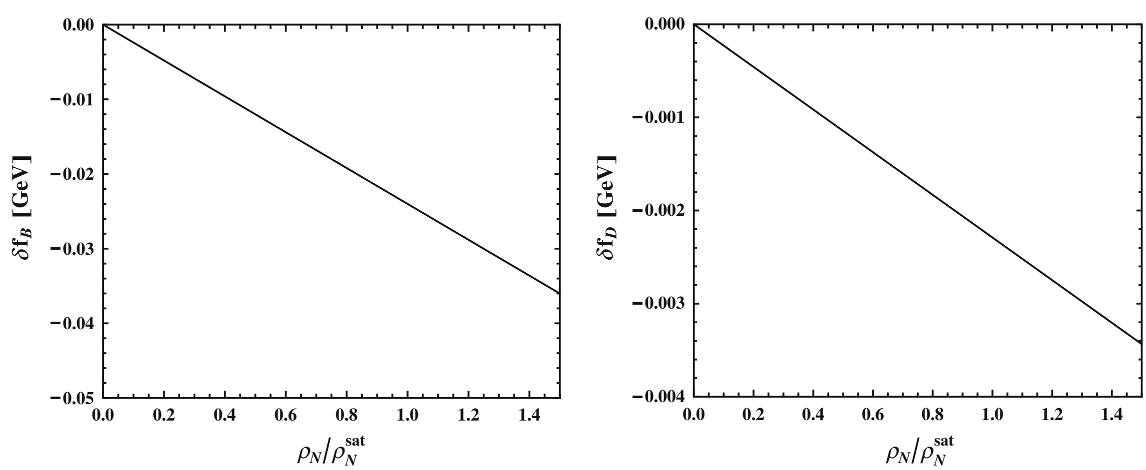

Fig. 3 The dependence of the shift in the decay constant of the B meson to the nuclear matter density (*left panel*). The same, but for D meson (*right panel*)

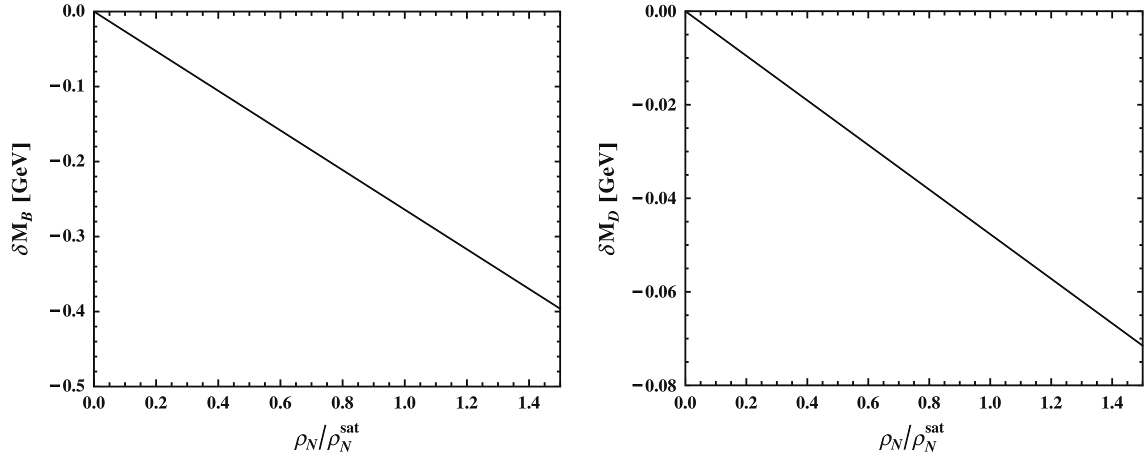

Fig. 4 The dependence of the shift in the mass of the B meson to the nuclear matter density (*left panel*). The same, but for D meson (*right panel*)

dictions of some existing theoretical works in the literature. We also discussed the dependence of the shifts in the decay constants and masses of these mesons on the nuclear matter density. The results obtained in the present work can be useful in analyzing the future experimental data at different heavy ion collision experiments. The results obtained for the shift in masses especially for those in the decay constants can also be used in theoretical calculations of the electromagnetic properties of the considered mesons as well as their strong couplings with other hadrons in nuclear medium.

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