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Properties of $D_2^*(2460)$ tensor meson and semileptonic transition of $B \to D_2^*(2460)\ell\bar{\nu}$

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Abstract

We employ QCD sum rules to calculate the leptonic decay constant of the $D_2^*(2460)$ tensor meson and the transition form factors of the semileptonic $B \to D_2^*(2460)\ell\bar{\nu}$ ($\ell = \tau, \mu, e$) decay channel. Using the fit functions of the form factors, we estimate the total decay widths and branching ratios of this transition at all lepton channels. The order of branching ratio indicates that this channel can be detected at LHC.

Keywords: QCD sum rules, leptonic decay constant, decay width, branching ratio

1. Introduction

The semileptonic $B$ meson transitions are promising frameworks to constrain the standard model parameters, determine the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and understand the origin of the CP violation. During the last few years, the orbitally excited charmed mesons have been attracted not only theoretically but also experimentally. After BaBar Collaboration reported their isolation of a number of orbitally excited charmed mesons in 2010 [1], the semileptonic decays of $B$ meson into the orbitally excited charmed meson have been in focus of attention of many theoretical works [2, 3, 4, 5]. One of the possible decay channels of the $B$ meson is its semileptonic transition to orbitally excited charmed mesons which provides a substantial contribution to the total semileptonic decay width. In this work, we calculate the leptonic decay constant of $D_2^*(2460)$ and the transition form factors of the semileptonic decay of $B \to D_2^*(2460)\ell\bar{\nu}$ in the framework of the QCD sum rules.

2. QCD sum rules for the leptonic decay constant of the $D_2^*(2460)$ tensor meson

In this section, the details of calculation for the leptonic decay constant of the $D_2^*(2460)$ tensor meson is presented. We start with the following two-point correlation function

$$\Pi_{\mu\nu,\alpha\beta} = i \int d^4x e^{iq(x-y)} \langle 0 | T [j_{\mu\nu}(x)\bar{j}_{\alpha\beta}(y)] | 0 \rangle, \quad (1)$$

where, $T$ is the time ordering operator. The interpolating current of the $D_2^*(2460)$ tensor meson, $j_{\mu\nu}$, can be written in terms of the quark fields as

$$j_{\mu\nu}(x) = \frac{i}{2} \left[ \bar{u}(x)\gamma_\mu D_\nu(x)c(x) + \bar{u}(x)\gamma_\nu D_\mu(x)c(x) \right], \quad (2)$$

where the $D_\mu(x)$ denotes the four-derivative with respect to $x$ acting on the left and right, simultaneously.
According to the general idea in the QCD sum rules, we need to calculate the aforementioned correlation function via two different ways: phenomenologically (physical side) and theoretically (QCD side). By equating these two representations, the QCD sum rules for the physical observables is obtained. To suppress the contributions of the higher states and continuum, Borel transformation as well as quark-hadron duality assumption are applied. As a result, the following sum rule for the leptonic decay constant of the \(D_s^*(2460)\) tensor meson is obtained (for more information see [6])

\[
f_{D_s^*}^2 e^{-\frac{m_D^2}{m_B^2} \langle \rho^{pert} + \rho^{pp} \rangle},
\]

where \(z_0\) is the continuum threshold and \(N\) is the Borel mass parameter. Since these parameters are auxiliary parameters, the physical quantities should be independent of them. As a result, the working regions for the Borel parameter and continuum threshold are found to be 3 GeV\(^2\) \(\leq N^2 \leq 6\) GeV\(^2\) and \(z_0 = 8.1 \pm 0.5\) GeV\(^2\), respectively. The perturbative and non-perturbative parts of the spectral densities are obtained as

\[
\rho^{pert} = \frac{N_c}{960 \pi^3 s} \left( \frac{m^2 - s}{2m^2 + 3s} \right)^4, \quad (4)
\]

and

\[
\rho^{pp} = -\frac{N_c}{48 \pi} m_c m_0^2 \langle \bar{u}u \rangle. \quad (5)
\]

From numerical analysis, the value of leptonic decay constant of the \(D_s^*(2460)\) tensor meson is obtained as \(f_{D_s^*(2460)} = 0.0228 \pm 0.0068\) at \(N^2 = 3\) GeV\(^2\) and \(z_0 = 8.2\) GeV\(^2\). Our result on the leptonic decay constant of the \(D_s^*(2460)\) meson can be check in the future experiments.

3. QCD sum rules for the form factors of the \(B \rightarrow D_s^*(2460)\ell \bar{\nu}\) transition

To derive the QCD sum rules for the form factors of the \(B \rightarrow D_s^*(2460)\ell \bar{\nu}\) transition, we start with the following tree-point correlation function

\[
\Pi_{\mu \alpha \beta}^{phys}(q^2) = \int d^4x \int d^4y \ e^{-ip\cdot x} e^{-ip\cdot y} 
(0 \ | \ T (J_{\mu \alpha \beta}^{D_s^*}(0) J_B^\mu(x) ) | 0), \quad (6)
\]

where, \(J^{\mu \alpha \beta}(0) = \bar{u}(0) \gamma_{\mu} (1 - \gamma_5) b(0)\) is the transition current. Also, the interpolating current of the \(B\) meson in terms of the quark fields is written as

\[
J_B(x) = \bar{u}(x) \gamma_5 \gamma_3 b(x). \quad (7)
\]

The interpolating current of \(D_s^*(2460)\) meson is defined in Eq (2). The aforesaid correlation function can be calculated in two different ways. In the physical side, the correlation function is obtained as following form

\[
\Pi^{phys}_{\mu \alpha \beta}(q^2) = \frac{\langle D_s^*(p', \epsilon) | J^\mu_{\alpha \beta}(0) | B(p) \rangle}{(p^2 - m_B^2)(p'^2 - m_{D_s^*}^2)} 
\times \langle 0 | J_{\alpha \beta}^{D_s^*}(0) | D_s^*(p', \epsilon) \rangle
\times \langle B(p) | J_B^\mu(0) | 0 \rangle + \cdots, \quad (8)
\]

where \(\cdots\) symbolizes contribution of the higher states and continuum, and \(\epsilon\) is the polarization vector of the \(D_s^*(2460)\) tensor meson. To proceed, we need to define the following matrix elements

\[
\langle 0 \ | \ J_{\alpha \beta}^{D_s^*}(0) | D_s^*(p', \epsilon) \rangle = m_{D_s^*}^3 \ f_{D_s^*} \epsilon_{\alpha \beta}
\]

\[
\langle B(p) | J_B^\mu(0) | 0 \rangle = -i m_{B}^{2} \ f_{B} \epsilon^\mu \epsilon^\nu \epsilon^\rho \epsilon^\sigma P_\sigma p_\rho P_\nu q_\mu
\]

\[
\langle D_s^*(p', \epsilon) | J_B^\mu(0) | B(p) \rangle = h(q^2) \epsilon_{\mu \nu \lambda \rho} \epsilon^{\nu \lambda \rho} P_\rho p_\nu q_\mu
\]

\[
\times \epsilon_{\mu \nu \lambda \rho} P_\rho p_\nu q_\mu + iK(q^2) \epsilon_{\mu \nu \lambda \rho} P_\rho p_\nu q_\mu \quad + \cdots, \quad (9)
\]

where \(h(q^2), b_+(q^2), b_-(q^2), K(q^2)\) are transition form factors; and \(f_{D_s^*}\) and \(f_B\) are leptonic decay constants of \(D_s^*\) and \(B\) mesons, respectively. Using Eq. (8) and Eq. (9), we obtain the final expression of physical side

\[
\Pi^{phys}_{\mu \alpha \beta}(q^2) = \frac{f_{D_s^*} f_B m_{D_s^*} m_B^2}{8(m_B^2 + m_u^2)(p^2 - m_B^2)} \left( p'^2 - m_{D_s^*}^2 \right)
\times \left[ \frac{2}{3} \Big( - \Delta K(q^2) + \Delta b_+(q^2) \Big) q_\mu g_{\beta \alpha}
\right.
\left. + \frac{2}{3} \Big( \Lambda - 4m_{D_s^*}^2 \Delta K(q^2) + \Lambda b_+(q^2) \Big) P_\mu g_{\beta \alpha}
\right.
\left. + i \left( \Delta - 4m_{D_s^*}^2 \right) h(q^2) \epsilon_{\mu \nu \lambda \rho} P_\rho P_\sigma q_\nu
\right.
\left. + i \left( \Delta - 4m_{D_s^*}^2 \right) h(q^2) \epsilon_{\mu \nu \lambda \rho} P_\rho P_\sigma q_\nu + \text{other structures} \right] + \cdots, \quad (10)
\]

where

\[
\Delta = m_B^2 - 2m_{D_s^*}^2 (m_{D_s^*}^2 + q^2) + (m_{D_s^*}^2 - q^2)^2
\]

\[
\Delta = m_D^2 + 5m_{D_s^*}^2 - q^2. \quad (11)
\]

To obtain the QCD side, we substitute the explicit forms of the interpolating currents into Eq. (6) and contract out all quark fields via the Wick’s theorem. Finally,
where $S^s_{ij}(x)$ and $S^s_{ij}(x)$ are the heavy and light quarks propagators. Replacing the expressions of the quarks propagators in Eq. (12) and performing integrals over $x$ and $y$ (for more information see [7]), we find the QCD side as

$$
\Pi^{\text{QCD}}_{\mu \nu}(q^2) = \frac{-i^3}{4} \int d^4x \int d^4y \, e^{-ip \cdot x} e^{ip' \cdot y} \times \left\{ T_r \left[ S^s_{ik}(x - y) \gamma_\alpha D_{\nu}(y) S^s_{ij}(y) \gamma_\mu \right] \times (1 - \gamma_5) S^s_{h}(x - i) \gamma_5 \beta \right\},
$$

(12)

where the perturbative part of correlation functions $\Pi^{\text{pert}}_{i}(q^2)$ are expressed in terms of double dispersion integrals in the following way

$$
\Pi^{\text{pert}}_{i}(q^2) = \int ds \int ds' \frac{\rho_i(s, s', q^2)}{(s - p^2)(s' - p'^2)},
$$

(14)

where $\rho_i(s, s', q^2) = \frac{1}{2} \text{Im} [\Pi^{\text{pert}}_{i}(q^2)]$ are the spectral densities. After lengthy calculations, we obtain the spectral densities. For instance for $\rho_1(s, s', q^2)$, we obtain

$$
\rho_1(s, s', q^2) = \int_0^1 dx \int_0^{1-x} dy \left\{ \frac{1}{64\pi^2 v^3} \times \left[ m_b v^3(8x^2 - 8y^2 + 6x - 6y - 6) + 3m_c(8x^2 + 6x^2(4y - 3) - 6x)(y - 1)^2 \right. \times (3 + 2y + 4y^2) - 2(3 + 2y + 4y^2) \times (y - 1)^3 + 2x^3(1 - 18y + 8y^2) \right. \\
+ \left. x^2(22 - 5y - 16y^2) \right]\},
$$

(15)

where $v = (x + y - 1)$ and $\Pi^{\text{nonpert}}_{1}(q^2)$ is

$$
\Pi^{\text{nonpert}}_{1}(q^2) = \left\{ \frac{m_b^4 + 4m_b^2m_c^2 + 2m_c^4(m^2_c - q^2)}{64r^2p^2} + \frac{(m_c^2 - q^2)^2}{64r^2p^2} + \frac{m_b^2m_c^2(m^2_c + m^2_c - q^2)}{32r^2p^2} \\
+ \frac{m_b^2m_c^2(m^2_c + m^2_c - q^2)}{32r^2p^2} + \frac{m_b^2m_c^2(m^2_c + m^2_c - q^2)}{32r^2p^2} + \frac{m_b^2m_c^2(m^2_c + m^2_c - q^2)}{32r^2p^2} \right\},
$$

(17)

$$
\times \left\{ \frac{-m_c^2 + 4m_b m_c + m_c^2 - q^2}{64r^2p^2} + \frac{m_b^2 + 2m_b^2m_c^2 + m_b^2m_c^2 - m_b^2q^2}{32r^2p^2} + \frac{m_b^2 + 2m_b^2m_c^2 + m_b^2m_c^2 - m_b^2q^2}{32r^2p^2} \right\}.
$$

(16)

where $r = p^2 - m_b^2$ and $r' = p'^2 - m_c^2$.

After acquiring the correlation function for both phenomenological and QCD sides, the coefficients of the same structures from both sides of the correlation functions are matched. In order to suppress the contribution of the higher states and continuum, we apply double Borel transformation with respect to initial and final momenta squared as well as the quark-hadron duality assumption. After these procedures, we get the sum rules for the form factors, such that $K(q^2)$ is

$$
K(q^2) = \frac{8(m_b + m_c)v^2}{f_{L}f_{D}m_{D}(m^2_b q^2 - m^2_b - 3m^2_{b}m^2_{D_{1}})} \times \left\{ \left( \int_{m_b^2}^{V_0} dx \int_{m_b^2}^{V_1} dy \int_{m_b^2}^{V_1} dx \int_{m_b^2}^{V_1} dy \right) \frac{v^2}{e^{\frac{m_b^2}{2} - \frac{m_b^2}{2}}} \rho_1(s, s', q^2) l_{L}(s, s', q^2) \right\}
$$

$$
+ \frac{v^2}{e^{\frac{m_b^2}{2} - \frac{m_b^2}{2}}} \left( \frac{m_b^2}{16} + 2 \left( \frac{3m_b^2}{M^2} + 2m_b m_c + 3m_c^2 - 3q^2 \right) \right)
$$

$$
+ \frac{m_b^2}{M^2 - q^2} - \frac{3m_b^2}{M^2 - q^2} - \frac{m_b^2}{M^2 - q^2} + \frac{m_b^2}{M^2 - q^2} - \frac{m_b^2}{M^2 - q^2}
$$

$$
+ \frac{m_b^2}{M^2 - q^2} - \frac{m_b^2}{M^2 - q^2} + \frac{m_b^2}{M^2 - q^2} + \frac{m_b^2}{M^2 - q^2} + \frac{m_b^2}{M^2 - q^2}
$$

where $r = p^2 - m_b^2$ and $r' = p'^2 - m_c^2$.
where
\[
L(s, s', q^2) = s'x - s x^2 - m_c^2 x - m_b^2 y + s x y \\
+ q^2 (x y - s x y - s' x y - s y^2).
\] (18)

The sum rules for the form factors contain four nonphysical auxiliary parameters, namely the Borel mass parameters \(M^2\) and \(M'^2\) and continuum thresholds \(s_0\) and \(s'_0\). We proceed to find their working regions at which the dependence of form factors on these parameters are weak. Our calculations show that in the intervals \(31 \text{ GeV}^2 \leq s_0 \leq 35 \text{ GeV}^2\) and \(7 \text{ GeV}^2 \leq s'_0 \leq 9 \text{ GeV}^2\) for continuum thresholds and \(10 \text{ GeV}^2 \leq M^2 \leq 20 \text{ GeV}^2\) and \(5 \text{ GeV}^2 \leq M'^2 \leq 15 \text{ GeV}^2\) for Borel mass parameters, our results weakly depend on these parameters. As an example, the dependence of the form factor \(K(q^2)\) on Borel mass parameter \(M^2\) is presented in figure 1. It can be seen from this figure that the form factor \(K(q^2)\) weakly depend on the \(M^2\) in its working region.

Our numerical analyzes indicate that the form factors are truncated at \(q^2 \approx 5 \text{ GeV}^2\). So, we need to obtain the fit functions of the form factors in the whole physical region to estimate the decay width of the \(B \rightarrow D_s^*(2460)\ell\bar{\nu}\) transition. The form factors are well fitted to the following fit function
\[
f(q^2) = f_0 \exp\left[c_1 \frac{q^2}{m_B^2} + c_2 \left(\frac{q^2}{m_B^2}\right)^2\right]
\] (19)
where the values of the parameters \(f_0, c_1\) and \(c_2\) obtained using \(M^2 = 15 \text{ GeV}^2\) and \(M'^2 = 5 \text{ GeV}^2\) for \(B \rightarrow D_s^*(2460)\ell\bar{\nu}\) decay are presented in the Table 1.

Our final purpose is to calculate the decay width and branching ratio of the \(B \rightarrow D_s^*(2460)\ell\bar{\nu}\) transition, presented in Table 2.

We expect that this transition can be detected at LHCb for all lepton channels. Any measurement on the form factors as well as decay rate of this channel and their comparison with our results can give more information about the nature and internal structure of the \(D_s^*(2460)\) tensor meson.

**References**


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**Table 1:** Parameters appearing in the fit function of the form factors.

<table>
<thead>
<tr>
<th>(K(q^2))</th>
<th>(f_0)</th>
<th>(c_1)</th>
<th>(c_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b_-(q^2))</td>
<td>(0.007 \pm 0.002 \text{ GeV}^{-2})</td>
<td>(0.14 \pm 0.04)</td>
<td>(10.70 \pm 0.82)</td>
</tr>
<tr>
<td>(b_+(q^2))</td>
<td>(-0.03 \pm 0.01 \text{ GeV}^{-2})</td>
<td>(1.20 \pm 0.15)</td>
<td>(22.52 \pm 1.68)</td>
</tr>
<tr>
<td>(h(q^2))</td>
<td>(-0.010 \pm 0.003 \text{ GeV}^{-2})</td>
<td>(1.19 \pm 0.13)</td>
<td>(1.12 \pm 0.08)</td>
</tr>
</tbody>
</table>

**Table 2:** Numerical results for decay widths and branching ratios at different lepton channels.

<table>
<thead>
<tr>
<th>(\Gamma(\text{GeV}))</th>
<th>(Br)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B \rightarrow D_s^*\tau\bar{\nu}_\tau)</td>
<td>((6.52 \pm 2.20) \times 10^{-17})</td>
</tr>
<tr>
<td>(B \rightarrow D_s^*\mu\bar{\nu}_\mu)</td>
<td>((4.04 \pm 1.18) \times 10^{-16})</td>
</tr>
<tr>
<td>(B \rightarrow D_s^*e\bar{\nu}_e)</td>
<td>((4.05 \pm 1.19) \times 10^{-16})</td>
</tr>
</tbody>
</table>

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**Figure 1:** \(K(q^2 = 0)\) as a function of the Borel mass \(M^2\) at fixed values of the \(s_0, s'_0\) and \(M'^2\).